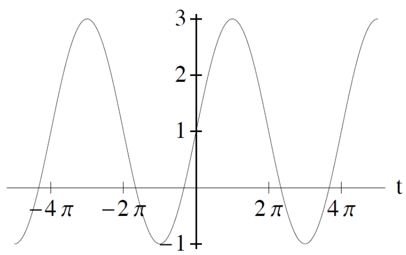
1. Consider the trigonometric function $f(t)$ whose graph is shown below. Write down a possible formula for $f(t)$.



Answer: This function appears to be an odd, periodic function that has been shifted upwards, so we will use $\sin(t)$ as our parent function and apply the appropriate transformations. The graph appears to be a function whose amplitude is 2, period is 4π, and midline is 1. So $f(t) = 2\sin\left(\frac{t}{2}\right) + 1$.

2. Find the domain of the function $f(x) = \sqrt{4 - 5x}$.

Answer: Because the square root of a negative number doesn't produce real numbers, we will only allow our domain to be values of x such that $4 - 5x$ is not negative. In other words, $4 - 5x \geq 0$. We solve this inequality for x:

$$\begin{aligned} 4 - 5x &\geq 0 \\ -5x &\geq -4 \\ x &\leq \frac{4}{5} \end{aligned}$$

So the domain is $(-\infty, 4/5]$.

3. What is the domain and range of the function $f(x) = (x + 3)^2 - 4$?

Answer: This function is a polynomial, so its domain is all real numbers. The shape of the graph is a parabola that opens upwards, so to find the range we first find the vertex. The equation of $f(x)$ is already in vertex form, so we know the vertex is $(-3, -4)$. Since the graph has a minimum at its vertex and extends upwards infinitely, the range is $[-4, \infty)$.

4. Use the limit definition of derivative to compute the derivative of the function $f(x) = \frac{4}{x}$ at $x = 2$.

Answer: The limit definition of a derivative is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Using this definition,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h} && \text{[Since } f(x) = \tfrac{4}{x}\text{]} \\
&= \lim_{h\to 0} \frac{\frac{4}{x+h}\left(\frac{x}{x}\right) - \frac{4}{x}\left(\frac{x+h}{x+h}\right)}{h} && \text{[Common denominators]} \\
&= \lim_{h\to 0} \frac{\frac{4x}{x(x+h)} - \frac{4(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{4x-4(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{4x-4x-4h}{x(x+h)}}{h} && \text{[Distributive property]} \\
&= \lim_{h\to 0} \frac{\frac{-4h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{-4h}{x(x+h)}\left(\frac{1}{h}\right) && \text{[Dividing by } h \text{ is the same as multiplying by } \tfrac{1}{h}\text{]} \\
&= \lim_{h\to 0} \frac{-4h}{x(x+h)h} \\
&= \lim_{h\to 0} \frac{-4}{x(x+h)} && \text{[Cancellation of } h\text{]} \\
&= \frac{-4}{x(x+0)} && \text{[Evaluate limit by setting } h = 0\text{]} \\
&= -\frac{4}{x^2} && \text{[Evaluate limit by setting } h = 0\text{]}
\end{aligned}
$$

The derivative at $x = 2$ is $f'(2) = -\frac{4}{(-2)^2} = -1$.

5. The table below gives the depth of snow that has fallen in inches as a function of time in hours past 8am. What is $d(1.5)$ (including units) and what does it represent? What time did it start snowing? Next, estimate $d'(1)$ and $d'(2)$. Include units in your answer and say in a full English sentence what the meaning of these numbers are.

x	.75	1	1.25	1.5	1.75	2	2.25	2.5
$d(x)$	0	.3	.8	1.25	1.85	2.0	2.2	2.4

Answer: $d(1.5) = 1.25$ inches, which represents the depth of the snow at 1.5 hours past 8 am, (which is 9:30 am). Since $d(.75) = 0$ inches, and after that time the depth of snow is positive, we can conclude that it started snowing between 8:45 am and 9:00 am. We can estimate $d'(1)$ by calculating the slope between $x = 1$ and $x = .75$:

$$\frac{d(1) - d(.75)}{1 - .75} = \frac{0.3}{0.25} = 1.2 \text{ inches/hr.}$$

We can estimate $d'(2)$ in the same way, by calculating the slope between $x = 2$ and $x = 1.75$:

$$\frac{d(2) - d(1.75)}{2 - 1.75} = \frac{2 - 1.85}{0.25} = 0.6 \text{ inches/hr.}$$

The interpretation is that $d'(1) \approx 1.2$ inches/hr means that at 9 am, the depth of the snow was increasing at 1.2 inches per hour. Likewise, $d'(2) \approx 0.6$ inches/hour means at 10 am, the depth of the snow was increasing at .06 inches per hour.

6. Say $f(x) = 3x^2 + x$. Use the definition of derivative to find $f'(1)$. Then write the equation of the tangent line to the curve at $x = 1$.

Answer:

$$\begin{aligned} f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} &= \lim_{h\to 0} \frac{3(1+h)^2 + (1+h) - (3(1)^2 + 1)}{h} \\ &= \lim_{h\to 0} \frac{3 + 6h + 3h^2 + 1 + h - 3 - 1}{h} \\ &= \lim_{h\to 0} \frac{7h + 3h^2}{h} \\ &= \lim_{h\to 0} 7 + 3h = 7. \end{aligned}$$

$f'(1) = 7$ is the slope of the tangent line at $x = 1$, and $f(1) = 4$, so the equation of the tangent line to the curve is $y - 4 = 7(x - 1)$.

7. Say that the position of an object moving horizontally is given by $s(t) = \sqrt{6t+7}$, where position is measured in miles and time is measured in hours. Find the instantaneous velocity at an arbitrary $t = a$, and then the instantaneous velocity at $t = 7$. Include units.

Answer: The instantaneous velocity at a time t is given by $s'(t)$,

$$\begin{aligned}
s'(a) = \lim_{h\to 0}\frac{s(a+h)-s(a)}{h} &= \lim_{h\to 0}\frac{\sqrt{6(a+h)+7}-\sqrt{6a+7}}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{6(a+h)+7}-\sqrt{6a+7}}{h}\left(\frac{\sqrt{6(a+h)+7}+\sqrt{6a+7}}{\sqrt{6(a+h)+7}+\sqrt{6a+7}}\right)\\
&= \lim_{h\to 0}\frac{6(a+h)+7-(6a+7)}{h(\sqrt{6(a+h)+7}+\sqrt{6a+7})}\\
&= \lim_{h\to 0}\frac{6h}{h(\sqrt{6(a+h)+7}+\sqrt{6a+7})}\\
&= \lim_{h\to 0}\frac{6}{(\sqrt{6(a+h)+7}+\sqrt{6a+7})}\\
&= \lim_{h\to 0}\frac{6}{2\sqrt{6a+7}}\\
&= \frac{3}{\sqrt{6a+7}}. \qquad \text{[This is instantaneous velocity at } t=a\text{]}
\end{aligned}$$

$s'(7) = \frac{3}{\sqrt{6(7)+7}} = \frac{3}{7}$ miles/hour. [This is instantaneous velocity at $t = 7$]

8. Consider the function

$$f(x) = \begin{cases} x^2+\cos x & \text{when } -5 < x < 0 \\ 5 & \text{when } x = 0 \\ e^{3x/2} & \text{when } 0 < x < 5 \end{cases}$$

(a) Define what it means for a function $f(x)$ to be continuous at a point $x = a$.
Answer: A function $f(x)$ is continuous at a point $x = a$ when $\lim_{x\to a} f(x)$ exists and it is equal to $f(a)$.

(b) $\lim_{x\to 2^+} f(x) =$ **Answer:** $\lim_{x\to 2^+} e^{3x/2} = e^3$.

(c) $\lim_{x\to 0^+} f(x) =$ **Answer:** $\lim_{x\to 0^+} e^{3x/2} = e^0 = 1$.

(d) $\lim_{x\to 0^-} f(x) =$ **Answer:** $\lim_{x\to 0^-} x^2+\cos x = 0^2+\cos 0 = 1$.

(e) $\lim_{x\to 0} f(x) =$ **Answer:** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1$.

(f) Is $f(x)$ continuous at $x = 0$? Fully explain your answer. If it is not continuous, state what type of discontinuity it is and why.
Answer: The limit as x approaches zero exists (we calculated it in part (e)), but it is not equal to $f(0)$. Therefore, $f(x)$ is not continuous at $x = 0$. Note that the discontinuity at $x = 0$ is a removable discontinuity because the $\lim_{x\to 0} f(x)$ exists.

9. Sketch the graph of a function $g(x)$ that satisfies ALL of the following properties:

   (a) $\lim_{x\to 1^+} g(x) = 2$

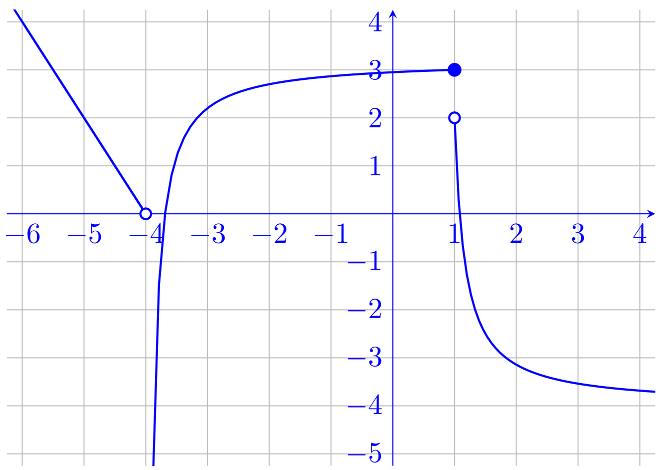
   (b) $\lim_{x\to 1} g(x)$ does not exist.

   (c) $\lim_{x\to \infty} g(x) = -4$

   (d) $\lim_{x\to -4^+} g(x) = -\infty$

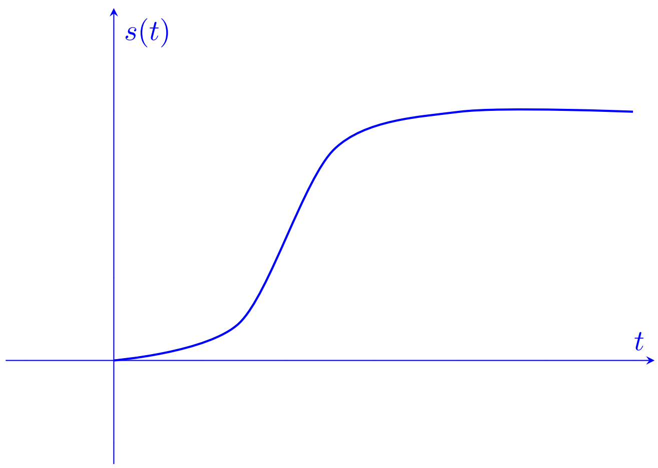
   (e) $\lim_{x\to -\infty} g(x) = \infty$

   Answer:



10. A car is first driven at an increasing speed and then the speed begins to decrease until the car stops. Sketch a graph of the distance the car has traveled as a function of time.

Answer: Assuming that the car is moving in the forward direction, when the car's speed is increasing, this means the velocity increasing. That implies that the second derivative is positive, so the graph of position should be concave up. Similarly, whenthe speed is decreasing, the velocity is decreasing. That implies that the second derivative is negative, so the graph of position is concave down. Then notice that when the car stops, position should become constant. The graph could look like this:



11. Find the average velocity over the interval $0.2 \leq t \leq 0.3$ of a car whose position $s(t)$ is given by the following table. Then estimate the velocity at $t = 0.3$. Include units.

t (sec)	0.1	0.2	0.3	0.4	0.5
$s(t)$ (ft)	0	0.5	1.0	1.8	2.8

Answer: The average velocity between $t = 0.2$ and $t = 0.3$ is

$$\frac{s(0.3) - s(0.2)}{0.3 - 0.2} = \frac{1 - 0.5}{0.1} = 5 \text{ ft/sec.}$$

To estimate the velocity at $t = 0.3$ better, let us also calculate the average velocity between $t = 0.3$ and $t = 0.4$ and then average the two average velocities. The average velocity between $t = 0.3$ and $t = 0.4$ is

$$\frac{s(0.4) - s(0.3)}{0.4 - 0.3} = \frac{1.8 - 1}{0.1} = 8 \text{ ft/sec.}$$

Then taking the average, $\frac{5 + 8}{2} = 6.5$ ft/sec, so 6.5 feet per second is an estimate for the velocity at $t = 0.3$.

12. The function $f(t) = -16t^2 + 64t$ gives the distance above the ground of a ball that is thrown from ground level straight up into the air at time $t = 0$, with an initial velocity of 64 ft/sec.

    (a) How high is the ball above the ground at $t = 1$ second?

    Answer: $f(1) = -16(1)^2 + 64(1) = 48$ ft.

    (b) How fast is the ball moving at $t = 1$, $t = 2$, $t = 3$ and $t = 4$ seconds?

    Answer: First we calculate the derivative, $f'(t)$ by using the limit definition. After we do this, we should get that $f'(t) - 32t + 64$. To calculate the velocity at $t = 1$, $t = 2$, $t = 3$, and $t = 4$, we substitute these values into $f'(t)$.

    $f'(1) = -32 + 64 = 32$ ft/sec
    $f'(2) = -32(2) + 64 = 0$ ft/sec
    $f'(3) = -32(3) + 64 = -32$ ft/sec
    $f'(4) = -32(4) + 64 = 64$ ft/sec.

    (c) What is the maximum height of the ball? (Hint: you should have gotten a velocity of 0 for one of the values of t in the last part)

    Answer: Since the ball travels upward and then starts to fall, we see that the maximum occurs when the ball has zero velocity at the peak of its trajectory. At time $t = 2$, the velocity is zero, so this must be the point at which the ball stops traveling upward (i.e. positive velocity) and begins its traveling downward (i.e. negative velocity). The maximum height is the height at $t = 2$, so $f(2) = -16(2)^2 + 64(2) = 64$ ft.

13. Precisely state the Intermediate Value Theorem. A polynomial $p(x)$ has $p(-2) = 13$ and $p(0) = -1$ and $p(1) = 1$. Show that $p(x)$ has at least two zeroes.

    Answer: IVT: If $f(x)$ is continuous on a closed interval $[a, b]$ and N is any number between $f(a)$ and $f(b)$ (where $f(a) \neq f(b)$), then there is a number c in (a, b) such that $f(c) = N$.

    Since $p(x)$ is polynomial, it is continuous everywhere. Thus we may use the IVT.

    $p(-2) = 13$ and $p(0) = -1$. Since 0 is between 13 and -1, the IVT states that there must exist a value c_1 between -2 and 0 such that $p(c_1) = 0$. So $p(x)$ has a zero between -2 and 0.

    Similarly, we know $p(0) = -1$ and $p(1) = 1$. Again, 0 is between -1 and 1, so there must exist a value c_2 between 0 and 1 such that $p(c_2) = 0$. So $p(x)$ has a zero between 0 and 1. Thus $p(x)$ has at least two zeros.

14. Find the following limits. Show all of your work.

(a) $\displaystyle\lim_{x\to 2}\frac{x^2+4x-12}{x-2}$

Answer:

$$\begin{aligned}\lim_{x\to 2}\frac{x^2+4x-12}{x-2} &= \lim_{x\to 2}\frac{(x-2)(x+6)}{(x-2)}\\ &= \lim_{x\to 2}(x+6)\\ &= 8\end{aligned}$$

(b) $\displaystyle\lim_{x\to 9}\frac{3-\sqrt{x}}{x-9}$

Answer:

$$\begin{aligned}\lim_{x\to 9}\frac{3-\sqrt{x}}{x-9} &= \lim_{x\to 9}\frac{(3-\sqrt{x})}{(x-9)}\frac{(3+\sqrt{x})}{(3+\sqrt{x})}\\ &= \lim_{x\to 9}\frac{9-x}{(x-9)(3+\sqrt{x})}\\ &= \lim_{x\to 9}\frac{-1}{3+\sqrt{x}}\\ &= -\frac{1}{6}\end{aligned}$$

(c) $\displaystyle\lim_{x\to 4}\frac{\frac{2}{x}-\frac{1}{2}}{x-4}$

Answer:

$$\begin{aligned}\lim_{x\to 4}\frac{\frac{2}{x}-\frac{1}{2}}{x-4} &= \lim_{x\to 4}\frac{(\frac{2}{x}-\frac{1}{2})}{(x-4)}\frac{(2x)}{(2x)}\\ &= \lim_{x\to 4}\frac{4-x}{(x-4)(2x)}\\ &= \lim_{x\to 4}\frac{-1}{2x}\\ &= -\frac{1}{8}\end{aligned}$$

(d) $\lim_{x\to\infty} \frac{3x^4+2x-1}{4-x^4}$

Answer:

$$\begin{aligned}
\lim_{x\to\infty} \frac{3x^4+2x-1}{4-x^4} &= \lim_{x\to\infty} \frac{(3x^4+2x-1)}{(4-x^4)}\frac{(\frac{1}{x^4})}{(\frac{1}{x^4})} \\
&= \lim_{x\to\infty} \frac{3+\frac{2}{x^3}-\frac{1}{x^4}}{\frac{4}{x^4}-1} \\
&= \frac{3+0+0}{0-1} \\
&= -3
\end{aligned}$$

(e) $\lim_{x\to\infty} \frac{3x^2+2x-1}{4-x}$

Answer:

$$\begin{aligned}
\lim_{x\to\infty} \frac{3x^2+2x-1}{4-x} &= \lim_{x\to\infty} \frac{(3x^2+2x-1)}{(4-x)}\frac{(\frac{1}{x})}{(\frac{1}{x})} \\
&= \lim_{x\to\infty} \frac{3x+2-\frac{1}{x}}{\frac{4}{x}-1} \\
&= -\infty
\end{aligned}$$

(f) $\lim_{x\to\infty} \frac{\sqrt{4x^2+2x-1}}{x+1}$

Answer:

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt{4x^2+2x-1}}{x+1} &= \lim_{x\to\infty} \frac{\sqrt{4x^2+2x-1}}{(x+1)}\frac{(\frac{1}{x})}{(\frac{1}{x})} \\
&= \lim_{x\to\infty} \frac{\sqrt{4x^2+2x-1}\sqrt{\frac{1}{x^2}}}{1+\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}}{1+\frac{1}{x}} \\
&= \frac{\sqrt{4+0-0}}{1+0} \\
&= 2
\end{aligned}$$

(g) $\lim_{x\to-\infty}(\sqrt{9x^2+x}-3x)$

Answer:

$$\begin{aligned}\lim_{x\to-\infty}(\sqrt{9x^2+x}-3x) &= \infty+\infty \\ &= \infty\end{aligned}$$

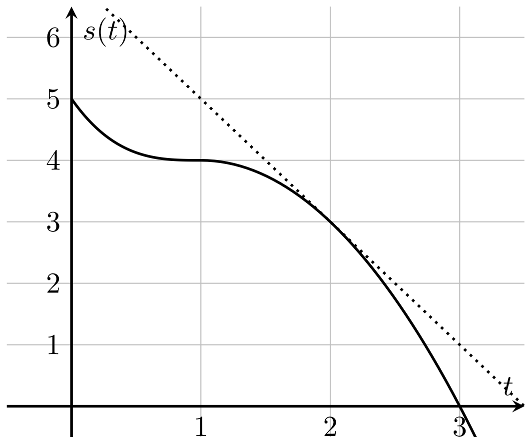
(Warning: While it is true that $\infty+\infty=\infty$, it is *not* true that $\infty-\infty=0$, Observe the following problem.)

(h) $\lim_{x\to\infty}(\sqrt{9x^2+x}-3x)$

Answer:

$$\begin{aligned}\lim_{x\to\infty}(\sqrt{9x^2+x}-3x) &= \lim_{x\to\infty}\frac{(\sqrt{9x^2+x}-3x)}{1}\frac{(\sqrt{9x^2+x}+3x)}{(\sqrt{9x^2+x}+3x)} \\ &= \lim_{x\to\infty}\frac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x} \\ &= \lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x} \\ &= \lim_{x\to\infty}\frac{x}{(\sqrt{9x^2+x}+3x)}\frac{(\frac{1}{x})}{(\frac{1}{x})} \\ &= \lim_{x\to\infty}\frac{1}{\sqrt{9+\frac{1}{x}}+3} \\ &= \frac{1}{\sqrt{9}+3} \\ &= \frac{1}{6}\end{aligned}$$

15. The curve below shows position s, measured in feet, as a function of time t, measured in seconds. The dotted line is tangent to the curve.



(a) Find the average velocity over the interval from $t = 0$ to $t = 3$. Include units.

Answer: average velocity $= \dfrac{s(3) - s(0)}{3 - 0} = \dfrac{0 - 5}{3} = -\dfrac{5}{3}$ ft/s

(b) Find the instantaneous velocity at $t = 2$. Include units.

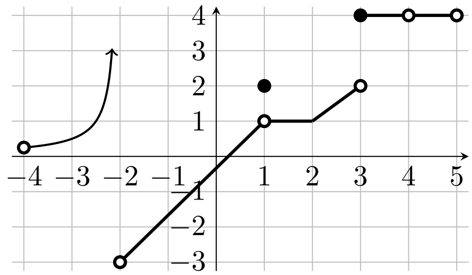
Answer: The instantaneous velocity is the slope of the tangent line at $t = 2$. This line goes through $(1, 5)$ and $(2, 3)$. So the instantaneous velocity is

$$s'(2) = \frac{3 - 5}{2 - 1} = -2 \text{ ft/sec.}$$

(c) Find the equation of the tangent line.

Answer: The equation of a line through $(2, 3)$ with slope -2 is $s - 3 = -2(t - 2)$ or $s = -2t + 7$.

16. Here's a graph of the function $f(x)$:



Meanwhile, here is a definition of the function $g(x)$:

$$g(x) = \begin{cases} -(x-3)^2 + 1 & x > 3 \\ -x + 6 & x < 3 \end{cases}$$

(a) Explain why the graph defines f as a function of x.

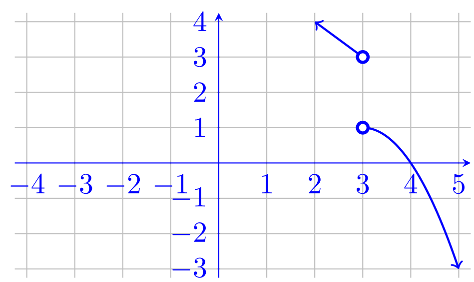
Answer: It passes the vertical line test.

(b) Is the f function invertible?

Answer: No, it fails the horizontal line test.

(c) Is the function g invertible?

Answer: By sketching g, we can see that it is inveritble.



(d) What is the domain and the range of f?

Answer: The domain of f is $(-4, -2) \cup (-2, 4) \cup (4, 5)$ and the range of f is $(-3, \infty)$.

(e) What is the domain and the range of g?

Answer: The domain of g is $(-\infty, 3) \cup (3, \infty)$ and the range of g is $(-\infty, 1) \cup (3, \infty)$.

(f) Find the limit of $f(x)$ as x approaches -2, as x approaches 1, as x approaches 2, as x approaches 3, as x approaches 4. Clearly explain your results.

Answer:

$$\lim_{x\to -2} f(x) = \text{DNE};\ \lim_{x\to -2^+} f(x) = -3 \text{ but } \lim_{x\to -2^-} f(x) = \infty$$
$$\lim_{x\to 1} f(x) = 1;\ \text{removable discontinuity}$$
$$\lim_{x\to 2} f(x) = 1; f \text{ is continuous at } 2$$
$$\lim_{x\to 3} f(x) = \text{DNE};\ \lim_{x\to 3^+} f(x) = 4 \text{ but } \lim_{x\to 3^-} f(x) = 2$$
$$\lim_{x\to 4} f(x) = 4;\ \text{removable discontinuity}$$

(g) What is the limit of $g(x)$ as x approaches 3? What about as x approaches 0?

Answer:

$$\lim_{x\to 3} g(x) = \text{DNE};\ \lim_{x\to 3^+} g(x) = 1 \text{ but } \lim_{x\to 3^-} g(x) = 3$$
$$\lim_{x\to 0} g(x) = 6; g \text{ is continuous at } 0$$

(h) List all of the discontinuities of f and g and clearly explain why these are discontinuities. State what type of discontinuities they are and why.

Answer: Discontinuities of f:

$$x = -4 : \lim_{x\to -4} f(x) \text{ exists, but } f(-4) \text{ is undefined, a removable discontinuity}$$
$$x = -2 : \lim_{x\to -2^-} f(x) = \infty,\ \text{an infinite discontinuity}$$
$$x = 1 : \lim_{x\to 1} f(x) = 1 \text{ but } f(1) = 2,\ \text{a removable discontinuity}$$
$$x = 3 : \lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x),\ \text{a jump discontinuity since both sided limits exist}$$
$$x = 4 : \lim_{x\to 4} f(x) = 4 \text{ but } f(4) \text{ is undefined, a removable discontinuity}$$
$$x = 5 : \lim_{x\to 5} f(x) \text{ exists, but } f(5) \text{ is undefined, a removable discontinuity}$$

Discontinuities of g:

$$x = 3 : \lim_{x\to 3^-} g(x) \neq \lim_{x\to 3^+} g(x),\ \text{a jump discontinuity since both sided limits exist}$$

(i) $\lim_{x\to 2}(f(x)+g(x)) =$

Answer:

$$\lim_{x\to 2}(f(x)+g(x)) = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 1+4 = 5$$

(j) $\lim_{x\to 3}(f(x)+g(x)) =$

Answer:

$$\lim_{x\to 3^+}(f(x)+g(x)) = \lim_{x\to 3^+} f(x) + \lim_{x\to 3^+} g(x) = 1+4=5$$
$$\lim_{x\to 3^-}(f(x)+g(x)) = \lim_{x\to 3^-} f(x) + \lim_{x\to 3^-} g(x) = 2+3=5$$
$$\lim_{x\to 3}(f(x)+g(x)) = 5$$

17. Find two values of the constant b so that the following function is continuous:

$$h(x) = \begin{cases} x^2+x-11 & x \geq b \\ -3x+1 & x < b \end{cases}$$

Answer: For $h(x)$ to be continuous, we need $\lim_{x\to b^+} h(x) = \lim_{x\to b^-} h(x)$. We have $\lim_{x\to b^+} h(x) = b^2+b-11$ and $\lim_{x\to b^-} h(x) = -3b+1$. Setting them equal gives,

$$\begin{aligned} & b^2+b-11 = -3b+1 \\ \Rightarrow\quad & b^2+4b-12 = 0 \\ \Rightarrow\quad & (b+6)(b-2) = 0 \\ \Rightarrow\quad & b=-6 \text{ or } b=2. \end{aligned}$$

18. If $4x-9 \leq f(x) \leq x^2-4x+7$ for $x \geq 0$, then find $\lim_{x\to 4} f(x)$.

Answer: By the Squeeze Theorem,

$$\lim_{x\to 4} 4x-9 \leq \lim_{x\to 4} f(x) \leq \lim_{x\to 4} x^2-4x+7$$
$$4(4)-9 \leq \lim_{x\to 4} f(x) \leq 4^2-4(4)+7$$
$$7 \leq \lim_{x\to 4} f(x) \leq 7.$$

Thus, $\lim_{x\to 4} f(x) = 7$.

19. Suppose $f(2)=3$ and $f'(2)=1$. Find $f(-2)$ and $f'(-2)$ if f is assumed to be even.

Answer: $f(-2)=3$ and $f'(-2)=-1$.

20. Given all of the following information about a function f, sketch its graph.

- $f(x) = 0$ at $x = -5$, $x = 0$, and $x = 5$
- $\lim_{x \to -\infty} f(x) = \infty$
- $\lim_{x \to \infty} f(x) = -3$
- $f'(x) = 0$ at $x = -3$, $x = 2.5$, and $x = 7$

Answer: There are many viable functions that can satisfy these conditions. Here is a rough sketch of one possible function: