EXAMPLES:

1. IS $f(x) = \begin{cases} 2x+1 & x \geq 3 \\ x^2+2 & x < 3 \end{cases}$

DIFFERENTIABLE AT $x = 3$?

FIRST CHECK FOR CONTINUITY:

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} 2x+1 = 7$

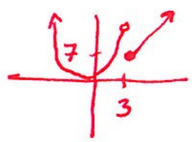
$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} x^2+2 = 11$

$\lim\limits_{x \to 3^+} f(x) \neq \lim\limits_{x \to 3^+} f(x)$, SO

$\lim\limits_{x \to 3} f(x)$ D.N.E.

SO $f(x)$ IS NOT CONINUOUS AT $x=3$,

SO $f(x)$ IS NOT DIFFERENTIABLE AT $x=3$



DISCONTINUITY $\Rightarrow$ NOT DIFFERENTIABLE

2. IS $f(x) = \begin{cases} x+1 & x \leq 0 \\ 1-x^2 & x > 0 \end{cases}$

DIFFERENTIABLE AT $x=0$?

AS ABOVE, CHECK FOR CONTINUITY:

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} 1-x^2 = 1$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x+1 = 1$

$\Rightarrow \lim\limits_{x \to 0} f(x) = 1$

$\lim\limits_{x \to 0} f(x) = f(0) = 1$. SO $f(x)$ IS CONTINUOUS

AT $x=0$. IT MIGHT BE DIFFERENTIABLE, IT MIGHT NOT BE... LETS CHECK:

$\lim\limits_{h \to 0^+} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^+} \frac{f(h)-1}{h} = \lim\limits_{h \to 0} \frac{1-h^2-1}{h}$

$= \lim\limits_{h \to 0} \frac{-h^2}{h} = \lim\limits_{h \to 0} -h = 0$

$\lim\limits_{h \to 0^-} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^-} \frac{f(h)-1}{h} = \lim\limits_{h \to 0^-} \frac{h+1-1}{h}$

$= \lim\limits_{h \to 0} \frac{h}{h} = \lim\limits_{h \to 0} 1 = 1$

$\lim\limits_{h \to 0^+} \frac{f(0+h)-f(0)}{h} \neq \lim\limits_{h \to 0^-} \frac{f(0+h)-f(0)}{h}$

SO $\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h}$ D.N.E. $f(x)$ IS

NOT DIFFERENTIABLE AT $x=0$.

CUSP $\Rightarrow$ NOT DIFFERENTIABLE

3. IS $f(x) = \begin{cases} 2x+1 & x < 0 \\ (x+1)^2 & x \geq 0 \end{cases}$

DIFFERENTIABLE AT $x=0$?

CHECK CONTINUITY, AS IN PREVIOUS PROBLEMS. IT IS CONTINUOUS.

TRY LIMITS OF DIFFERENCE QUOTIENTS FROM LEFT AND RIGHT.

$\lim\limits_{h \to 0^+} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^+} \frac{f(h)-f(0)}{h}$

$= \lim\limits_{h \to 0^+} \frac{(h+1)^2-1}{h} = \lim\limits_{h \to 0^+} \frac{h^2+2h+1-1}{h}$

$= \lim\limits_{h \to 0^+} \frac{\cancel{h}(h+2)}{\cancel{h}} = \lim\limits_{h \to 0} h+2 = 2$

$\lim\limits_{h \to 0^-} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^-} \frac{2h+1-1}{h} = \lim\limits_{h \to 0^-} \frac{2\cancel{h}}{\cancel{h}} = 2$

$\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h} = 2$, SO $f(x)$ IS DIFFERENTIABLE AT $x=0$.

JOIN IS SMOOTH, f is Differentiable

4. IS $f(x) = x^{\frac{1}{3}}$ DIFFERENTIABLE AT $x=0$?

$\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0} \frac{h^{1/3}-0}{h}$

$= \lim\limits_{h \to 0} \frac{h^{\frac{1}{3}}}{h} = \lim\limits_{h \to 0} \frac{1}{h^{2/3}} = \infty$

$f(x)$ IS NOT DIFFERENTIABLE AT $x=0$, IT HAS A VERTICAL TANGENT LINE