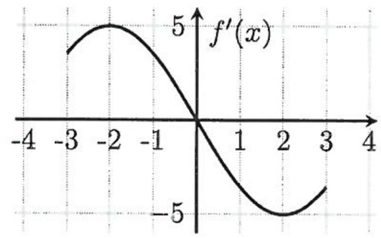
SOLUTIONS

MATH 1300

1. The graph of $f'(x)$, the derivative of $f(x)$, is shown below.



(a) Where does the absolute maximum of $f(x)$ on $(-3, 3)$ lie? Fully justify your answer.

critical numbers of f: $x=0$.
f' switches from + to − at $x=0$, so by the 1st derivative test, f has a local max at $x=0$.
f has only one critical number on $(-3,3)$, so f also has an absolute max at $x=0$.

(b) Where do the inflection points of $f(x)$ lie? Fully justify your answer.

From the graph, f' is increasing on $(-3,-2)$ and $(2,3)$, And decreasing on $(-2,2)$.

So f'' is positive on $(-3,-2)$ and $(2,3)$ and negative on $(-2,2)$.
f'' switches signs only at $x=-2$ and $x=2$, so the only inflection points of f are at $x=-2$ and $x=2$.

2. (a) Is $f(x) = 5 + 54x - 2x^3$ guaranteed to have an absolute maximum and absolute minimum on the interval $[0, 4]$? Explain in full English sentences.

$f(x)$ is a polynomial and thus continuous on the closed interval $[0,4]$. Thus by the E.V.T f is guaranteed to attain an absolute max and min on $[0,4]$

(b) Find the absolute maximum and absolute minimum of $f(x) = 5 + 54x - 2x^3$ on the interval $[0, 4]$ if they exist. Fully justify and explain your answers.

By the closed interval method, the absolute extrema can only occur at endpts and critical numbers.

$f'(x) = 54 - 6x^2 = 6(9 - x^2) = 6(3-x)(3+x)$.

critical numbers at $x = 3$, $\cancel{x = -3}$ ← not in interval.

x	$f(x)$
0	$\boxed{5}$
3	$\boxed{113}$
4	93

abs. min of $f(x)$ on $[0,4]$ is 5, occurs at $x=0$

abs. max of $f(x)$ on $[0,4]$ is 113, occurs at $x=3$

3. (a) Find and classify all critical numbers of $f(x) = 3xe^{-2x}$. Fully justify and explain your answers.

$f'(x) = 3x(-2e^{-2x}) + 3e^{-2x} = (-6x+3)e^{-2x}$

$f'(x) = 0$ when $-6x + 3 = 0$, $x = \frac{1}{2}$ is the only critical number.

$f'(x)$: + on the left of $\frac{1}{2}$, − on the right

$f'(0) = 3 > 0$ $\qquad$ $f'(1) = -3e^{-2} < 0$

f' switches from + to − at $x = \frac{1}{2}$, so by the first derivative test, f' has a local max at $x = \frac{1}{2}$

(b) Find the absolute maximum of $f(x) = 3xe^{-2x}$ for $x > 0$. Fully justify and explain your answers.

From above, $f(x)$ has a local max at $x = \frac{1}{2}$. There is only one critical number, so it is also an absolute max. The absolute max value is $f(\frac{1}{2}) = \frac{3}{2e}$