1. Consider the function $f(x) = (x-2)^3 - 3x + 5$ on the domain $[2, 5]$.

   (a) (4 points) Prove that $f(x)$ has an absolute maximum value on $[2, 5]$. (Hint: Do not perform any calculations.)

   (b) (8 points) At what x-value does $f(x)$ attain a maximum value on its domain? What is the maximum value?

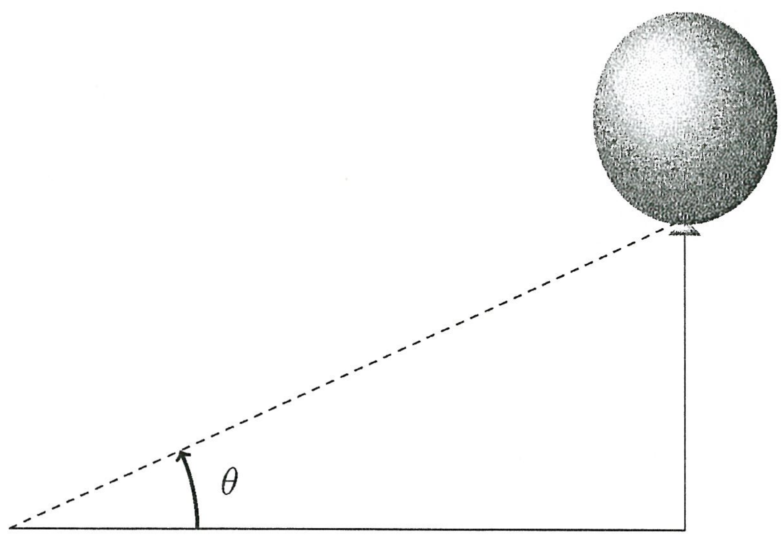
2. Compute the following limits:

(a) (3 points) $\lim_{x \to -1} \frac{3x^2 + 4x + 1}{x^3 - x + 8}$

(b) (3 points) $\lim_{x \to 1} \frac{x^3 - x^2 - x + 1}{x^3 - 4x^2 + 5x - 2}$

(c) (5 points) $\lim_{x \to 0+} x^{\frac{1}{x}}$

3. (12 points) An observer stands 200 m from the launch site of a hot-air balloon. The balloon rises vertically at a constant rate of 4 m/s. How fast is the angle of elevation of the balloon increasing 50 s after launch?



4. (16 points) A carpenter has been asked to build an open box with square base. The sides of the box will cost \$3 per square foot, and the base will cost \$4 per square foot. What are the dimensions of the box of greatest volume that can be constructed for \$48?

5. The function $f(x) = 2x^2$ satisfies the hypotheses of the Mean Value Theorem on the interval [0,2]. Find the number c that satisfies the conclusion of the Mean Value Theorem.

6. Find the inflection points (both x and y-coordinates) of $y = f(x) = 5x^4 - x^5$.

7. Use the Closed Interval Method to find the maximum and minimum values of $f(x) = x^3 - 12x - 1$ on the interval [-5, 4].

8. Suppose an object moves a long a straight line where its acceleration is given by $a = 3\cos t$. Find the velocity v and position s for the object's motion if it is known that $v(0) = 2$ and $s(0) = 4$.

PART 2. **Part 2 consists of 4 problems worth 13 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.**

1. Show how you use calculus to find two positive numbers whose product is 20 and whose sum is a minimum.

2. A rectangle has a perimeter (length around the outside) of 120 ft. Find the dimensions which make the area as large as possible.

Problem 3

Evaluate the following limits.

(a) $\lim_{x\to\infty} \dfrac{x+3}{2x-1}$

(b) $\lim_{x\to 0} \dfrac{x-\sin x}{x^2}$

Problem 4

A box with a *square base* and *open top* must have a volume of 4 ft^3. Find the *dimensions* of the box that minimize the amount of material used.

[Hint: The volume of a box is given by $V = x^2 h$, where x is the length of each edge of the square base and h is the height of the box. In this case, the area to be minimized is given by $A = x^2 + 4xh$ for an open top box.]

7.) A street light is mounted at the top of a 15ft. tall pole. A boy 5 ft. tall walks away from the pole at a speed of $3 ft/\sec$ along a straight path. How fast is the tip of his shadow moving when he is 40 ft. from the pole?

9.) Find $\lim_{x \to 0} \frac{\cos(7x) - \cos(3x)}{x^2}$

Part II

Part II consists of 3 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [**10 pts**] Find the absolute max/min of the function $f(x) = (x^2 - 2x)^3$ on the interval $[-2, 2]$.

(2) Given the function $f(x) = \frac{(x^2-4)}{(x+1)^2}$

(a) [**2 pts**] Find the x and y intercepts of the function.

(b) [**3 pts**] Find all asymptotes.

(c) **[4 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(d) **[2 pts]** Find the local maximum and local minimum value(s) of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).

(g) **[5 pts]** Use the above information to graph the function **on the next page**. Indicate all relevant information in the graph.