1. (a) (4 points) Explain why if $g(x)$ is a polynomial, it is guaranteed to have an absolute minimum and an absolute maximum on $[-1, 2]$.

POLYNOMIALS ARE CONTINUOUS FOR ALL X, AND IN PARTICULAR ON $[-1, 2]$. A CONTINUOUS FUNCTION ON A CLOSED INTERVAL LIKE $[-1, 2]$ IS GUARANTEED TO HAVE AN ABSOLUTE MAX AND MIN ON THAT INTERVAL.

(b) (10 points) Use the closed interval method to find the absolute maximum and absolute minimum of the function

$$g(x) = x^3 - 6x^2 + 5$$

on the interval $[-1, 2]$. Show your work.

$$g'(x) = 3x^2 - 12x$$
$$= 3x(x-4)$$
$$= 0 \text{ WHEN } x=0 \text{ AND } x=4,$$

BUT IGNORE $x=4$ SINCE 4 IS NOT IN $[-1,2]$.

CRIT NUM: $g(0) = 0^3 - 6 \cdot 0^2 + 5 = 5$ ← ABS MAX

ENDPTS: $g(-1) = (-1)^3 - 6(-1)^2 + 5 = -1 - 6 + 5 = -2$

$g(2) = 2^3 - 6(2^2) + 5 = 8 - 24 + 5 = -11$ ← ABS MIN

$g(x)$ HAS AN ABSOLUTE MINIMUM OF -11, OCCURING AT $x=2$

$g(x)$ HAS AN ABSOLUTE MAXIMUM OF 5, OCCURING AT $x=0$

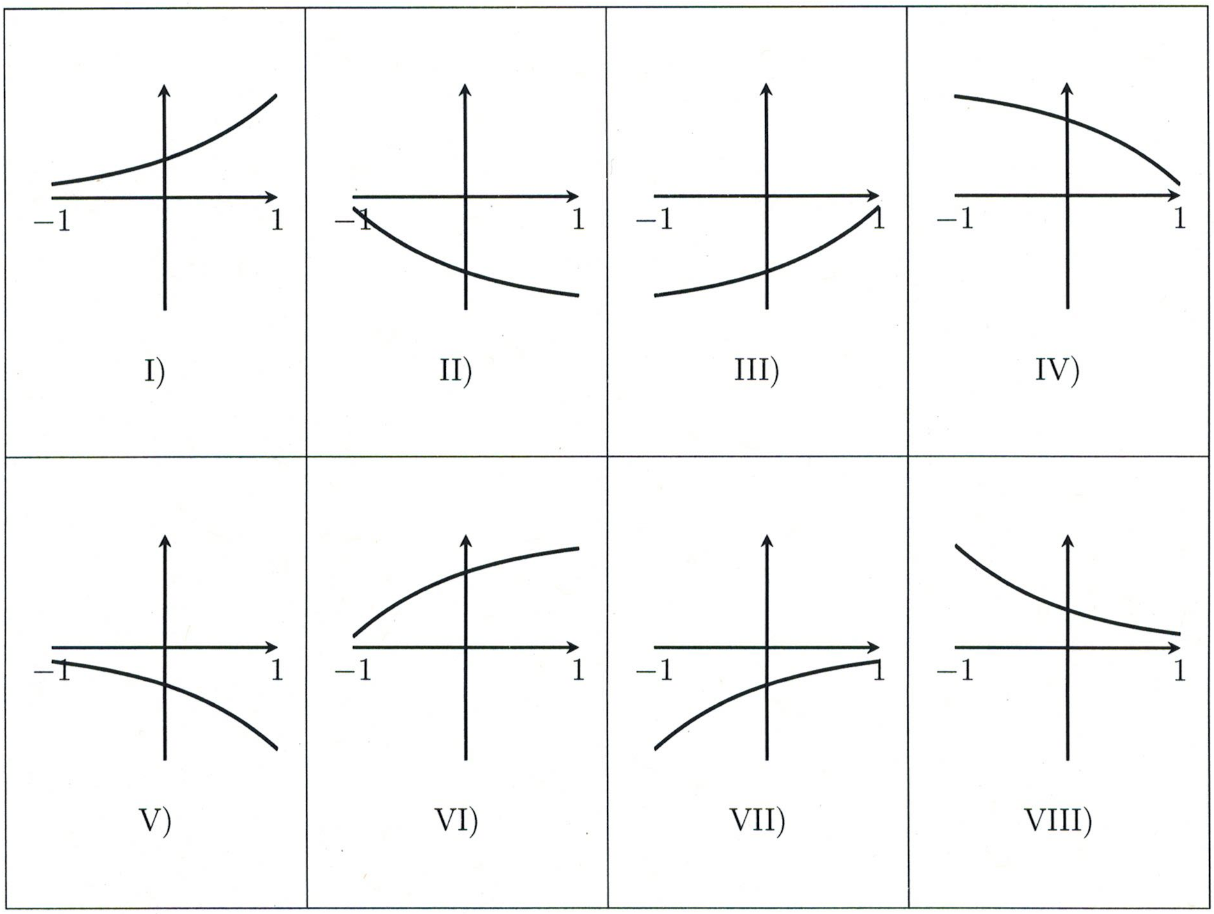
2. (12 points) For each of the following criteria, choose the graph of $f(t)$ which most accurately reflects the given information on the interval $[-1, 1]$.

a) $f(t)$ is negative
$f(t)$ is decreasing
$f''(t) > 0$: f CU
Graph: II

b) $f(t) < 0$
$f'(t) < 0$: f DECR
$f(t)$ is concave down
Graph: V

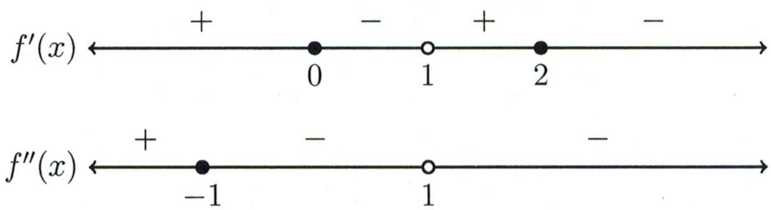
c) $f(t) > 0$
$f'(t) < 0$: f DECR
$f'(t)$ is decreasing : f CD
Graph: IV

d) $f(t)$ is positive
$f'(t)$ is positive : f INCR
$f'(t)$ is decreasing : f CD
Graph: VI



3. Consider the following information about a function $f(x)$ and the signs of its derivatives. *Note: A closed circle indicates that the corresponding derivative has a zero at that x-value, and an open circle indicates that the corresponding derivative is undefined at that x-value.*

$$\lim_{x \to 1} f(x) = f(1) = -1$$



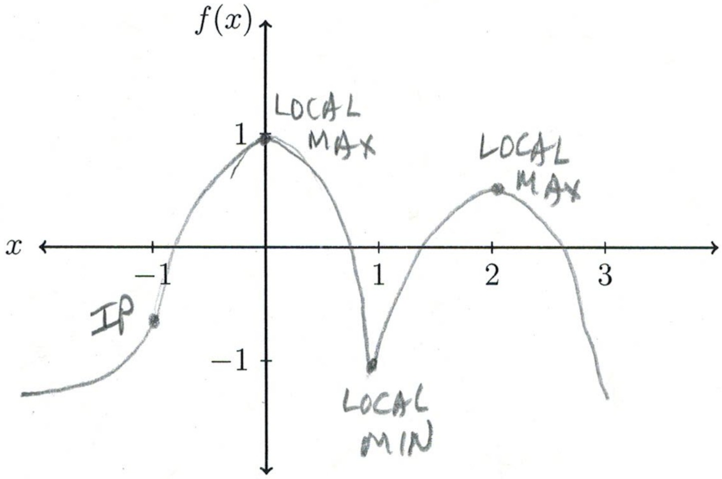
(a) (4 points) List the critical number(s) of $f(x)$ and classify each as a local minimum of $f(x)$, a local maximum of $f(x)$, or neither. Fully justify your answer.

CRITICAL NUMBERS: $X = 0, 1, 2$: $f'(x) = 0$ OR $f'(x) = $ DNE
LOCAL MAX AT $X = 0$: f' CHANGES FROM + TO −
LOCAL MIN AT $X = 1$: f' CHANGES FROM − TO +
LOCAL MAX AT $X = 2$: f' CHANGES FROM + TO −

(b) (4 points) At which x-value(s) does $f(x)$ have inflection point(s)? Fully justify your answer.

INFLECTION PT AT $X = -1$: f'' CHANGES FROM + TO −

(c) (6 points) Sketch a possible graph of $f(x)$ and mark all local extrema and inflection points.



4. (12 points) Circle the best answer for the value of each limit below. You do not need to show any work.

(a) $\lim_{x \to \infty} \frac{x^3}{e^x}$.

I) ∞ II) 0 III) 3 IV) 1 V) D.N.E.

$$\lim_{x \to \infty} \frac{x^3}{e^x} \stackrel{LH}{=} \lim_{x \to \infty} \frac{3x^2}{e^x} \stackrel{LH}{=} \lim_{x \to \infty} \frac{6x}{e^x} \stackrel{LH}{=} \lim_{x \to \infty} \frac{6}{e^x} = 0$$

(b) $\lim_{x \to 0} \frac{\cos(x)}{1 - \sin(x)}$.

I) 0 II) -1 III) D.N.E. IV) 1 V) ∞

$$\lim_{x \to 0} \frac{\cos x}{1 - \sin x} = \frac{\cos 0}{1 - \sin 0} = \frac{1}{1-0} = 1$$

(c) $\lim_{x \to \infty} \frac{x^7 + x^6 + 2x^5 + 3x^4 + 5x^3 + 8x^2 + 13x + 21}{12x^7}$. $= \frac{1}{12}$

I) ∞ II) 1 III) $\frac{1}{12}$ IV) 7 V) 0

DEGREE (NUMERATOR) = DEGREE (DENOMINATOR).

SO LIMIT AT ∞ IS RATIO OF LEADING COEFFICIENTS

$$\frac{1x^7}{12x^7} = \frac{1}{12}$$

5. (4 points) Circle an antiderivative of the function $f(x)$ from the choices given. You do not need to show any work.

$$f(x) = ax^3 + bx^2 + cx + d, \quad (a, b, c, d \text{ constants})$$

I) $3ax^2 + 2bx^2 + c$

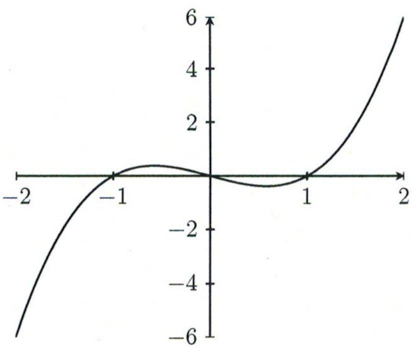
II) $\dfrac{ax^4}{4} + \dfrac{bx^3}{3} + \dfrac{cx^2}{2} + dx$

III) $3ax^4 + 3bx^3 + 2cx^2 + dx$

IV) $\dfrac{ax^2}{2} + \dfrac{bx}{2} + c$

6. (4 points) The function $f(x)$ is given by this graph:

$f(x)$:



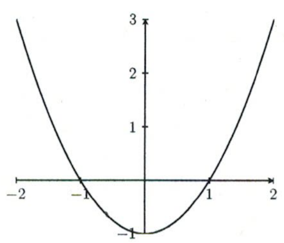
DIST UP/DOWN TO GRAPH

=

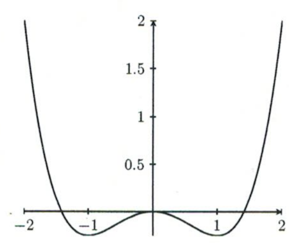
SLOPE OF TAN LINES

Circle an antiderivative of the function $f(x)$ from the choices given. You do not need to show any work.

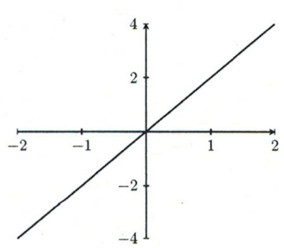
I)



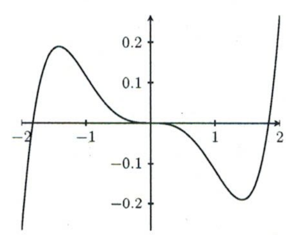
II)



III)



IV)



7. (12 points) Farmers can get 40 dollars per bale of hay on June 1, and after that, the price drops by 1 dollar per bale per extra day. On June 1, a farmer has 10 bales of hay in the field, and estimates that the crop is increasing at the rate of 1 bale per day. The farmer is trying to determine on which day to harvest the hay in the field.

(a) What is a formula for the price, $P(x)$, of a bale of hay, where x is the number of days after June 1?

$$P(X) = 40 - X$$

(b) What is a formula for the quantity of bales of hay, $Q(x)$, where x is the number of days after June 1?

$$Q(X) = 10 + X$$

(c) If the revenue function, $R(x)$, is defined as $R(x) = P(x)Q(x)$, when should the farmer harvest the hay to maximize the farmer's revenue?

$$R(X) = P(X)\,Q(X) = (40-x)(10+x)$$

$$R'(x) = (40-x)'(10+x) + (40-x)(10+x)'$$

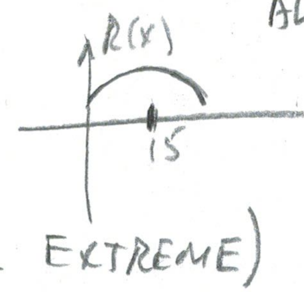
$$= (-1)(10+x) + (40-x)(1)$$

$$= 30 - 2X = 0 \text{ WHEN } \boxed{X = 15 \text{ DAYS AFTER JUNE } 1}$$

(d) Justify that the absolute maximum revenue occurs on the day you found above.

$$R''(x) = D_x[30-2x] = -2 < 0 \Rightarrow R(x) \text{ IS CD FOR ALL } X$$

SO LOCAL MAX OF $X=15$ IS AN ABSOLUTE MAX



(SINCE $X=15$ IS THE <u>ONLY</u> LOCAL EXTREME)

8. (10 points) Find $\lim_{x\to 0^+} 3x^{2x}$. Show your work.

Let $y = x^{2x}$

$$\lim_{x\to 0^+} 3x^{2x} = 3\cdot \lim_{x\to 0^+} x^{2x} = 3\cdot \lim_{x\to 0^+} y = 3e^0 = \boxed{3}$$

Let $y = x^{2x}$

$\ln y = \ln(x^{2x})$

$= 2x \cdot \ln x$

$$\lim_{x\to 0^+} (\ln y) = \lim_{x\to 0^+} 2x\cdot \ln x = 2\cdot\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$$

$$\stackrel{LH}{=} 2\cdot \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 2\cdot \lim_{x\to 0^+} \left(\frac{1}{x}\right)\left(-\frac{x^2}{1}\right)$$

$$= 2\cdot \lim_{x\to 0^+} (-x) = 2\cdot(-0) = 0$$

9. (8 points) Find the most general antiderivative for the function

$$h(x) = \frac{x+1}{x} + \frac{1}{6x^2} + \frac{1}{1+x^2}$$

$$\int h(x)\,dx = \int \frac{x+1}{x}\,dx + \int \frac{1}{6x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

$$= \int \left(1+\frac{1}{x}\right)dx + \frac{1}{6}\int x^{-2}\,dx + \tan^{-1}x + C_1$$

$$= x + \ln|x| + \frac{1}{6}\left(\frac{x^{-1}}{-1}\right) + \tan^{-1}x + C$$

$$= \boxed{x + \ln|x| - \frac{1}{6x} + \tan^{-1}x + C}$$

10. (10 points) Phillip went for a hike. He recorded his velocity at 15 minute intervals. Phillip created the following chart.

RECT 1 RECT 2 RECT 3 RECT 4

t (in hours)	0	.25	.5	.75	1	1.25	1.5	1.75	2
velocity (in miles per hour)	.5	.75	1.5	1	2	1.75	2	2.5	3

Approximate how far Phillip hiked in the 2 hours he recorded using 4 rectangles and right-hand endpoints. (In other words, use a right Riemann sum with $n = 4$.) Include units in your answer.

WITH $\Delta t = .5$:

$v(.5)\Delta t + v(1)\Delta t + v(1.5)\Delta t + v(2)\Delta t$ miles

$= \boxed{(1.5)(.5) + (2)(.5) + (2)(.5) + 3(.5) \text{ miles}}$

$= .5[1.5 + 2 + 2 + 3]$ miles

$= \boxed{.5[8.5] \text{ miles}}$