1. (12 points) Show your work. Let $f(x) = xe^x$.

a) Find the interval(s) where f is increasing and the interval(s) where f is decreasing.

$$f'(x) = x'e^x + x(e^x)'$$
$$= e^x + xe^x$$
$$= e^x(1+x)$$

$e^x > 0$ for all x

f INCR: $f'(x) > 0$ so $1+x > 0$ so $x > -1$ i.e. x IN $(-1, \infty)$

f DECR: $f'(x) < 0$ so $1+x < 0$ so $x < -1$ i.e. x in $(-\infty, -1)$

$f(x)$ is increasing on: $(-1, \infty)$

$f(x)$ is decreasing on: $(-\infty, -1)$

b) Find the interval(s) where f is concave up and the interval(s) where f is concave down.

$$f''(x) = [f'(x)]' = [e^x(1+x)]'$$
$$\overset{PROD}{=} (e^x)'(1+x) + e^x(1+x)'$$
$$= e^x(1+x) + e^x$$
$$= e^x(1+x+1) = e^x(x+2)$$

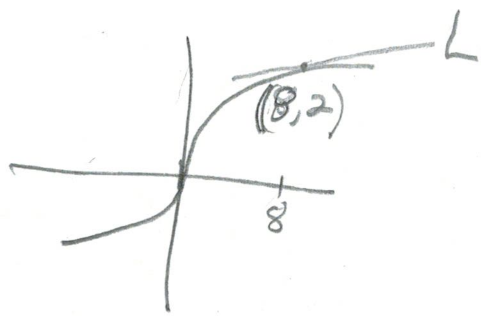
f CU: $f''(x) > 0$ so $x+2 > 0 \Rightarrow x > -2$ i.e. x in $(-2, \infty)$

f CD: $f''(x) < 0$ so $x+2 < 0 \Rightarrow x < -2$ i.e. x in $(-\infty, -2)$

$f(x)$ is concave up on: $(-2, \infty)$

$f(x)$ is concave down on: $(-\infty, -2)$

2. (10 points) Show your work. Let $f(x) = \sqrt[3]{x}$.



a) Find the linearization $L(x)$ of f at $a = 8$.

$$m_L = \left[D_x \sqrt[3]{x}\right]\Big|_{x=8} = \left[D_x\left(x^{1/3}\right)\right]\Big|_{x=8}$$

$$= \frac{1}{3}x^{-2/3}\Big|_{x=8} = \frac{1}{3}\cdot\frac{1}{8^{2/3}} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$$

LINE THRU (8,2) WITH SLOPE $m = \frac{1}{12}$:

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{1}{12}(x-8)$$

$$\boxed{y = L(x) = 2 + \frac{1}{12}(x-8)}$$

b) Use the linearization from part (a) to approximate $\sqrt[3]{7.7}$. Do **not** simplify.

$$\sqrt[3]{7.7} = f(7.7) \approx L(7.7) = \boxed{2 + \frac{1}{12}(7.7-8)}$$

$$\text{OR } = \boxed{2 + \frac{1}{12}(-.3)} = 1.975$$

CALCULATOR: 1.97468

COMPARE

3. (4 points) Multiple Choice: Find the slope of the line tangent to $y = \cos(3\theta)$ at $\theta = -\frac{\pi}{18}$. Circle the correct answer. You do **not** need to show work.

I) $\frac{3}{2}$ II) $\frac{3\sqrt{3}}{2}$ III) $\frac{\sqrt{3}}{2}$ IV) $-\frac{3}{2}$

SHOWING WORK ANYWAY SO YOU CAN SEE WHY...

$$m = \frac{d}{d\theta}\left[\cos(3\theta)\right]\Big|_{\theta=-\frac{\pi}{18}} = -\sin(3\theta)\cdot 3\Big|_{\theta=-\frac{\pi}{18}}$$

$$= -\sin\left[3\left(-\frac{\pi}{18}\right)\right]\cdot 3 = -\sin\left(-\frac{\pi}{6}\right)\cdot 3 = -\left(-\frac{1}{2}\right)\cdot 3$$

$$= \boxed{3/2}$$

4. (15 points) Multiple Choice: Differentiate the following functions. Circle the correct answer. You do **not** need to show work.

a) $a(x) = x^4 + 2x^3 + 6x$ $\quad a'(x) = 4x^3 + 6x^2 + 6$

I) $4x^4 + 6x^3 + 6x$ $\quad$ II) $4x^3 + 6x^2 + 6$ $\quad$ III) $4x^3 + 6x + 6$ $\quad$ IV) $4x^3 + 6x^2$

b) $b(x) = \sqrt{x} + \dfrac{1}{3x^2}$ $\quad b'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{3} D_x [x^{-2}] = \frac{1}{2\sqrt{x}} + \frac{1}{3}(-2x^{-3}) = \frac{1}{2\sqrt{x}} - \frac{2}{3x^3}$

I) $\dfrac{1}{2\sqrt{x}} + \dfrac{1}{6x}$ $\quad$ II) $\dfrac{1}{2\sqrt{x}} - \dfrac{2}{3x^3}$ $\quad$ III) $\dfrac{\sqrt{x}}{2} - \dfrac{2}{3x^3}$ $\quad$ IV) $\dfrac{1}{2\sqrt{x}} - \dfrac{6}{x^3}$

c) $c(x) = (x^3 - x)e^x$ $\quad c'(x) = (x^3-x)'e^x + (x^3-x)(e^x)' = (3x^2-1)e^x + (x^3-3)e^x$

I) $3x^2 - e^x$ $\quad$ II) $(3x^2 - 1)e^x$

III) $3x^2 - 1 \cdot e^x + x^3 - x \cdot e^x$ $\quad$ IV) $(3x^2 - 1)e^x + (x^3 - x)e^x$

d) $d(t) = \dfrac{t+2}{t+1}$

$$d'(t) = \frac{(t+1)\cdot(t+2)' - (t+2)(t+1)'}{(t+1)^2} = \frac{(t+1)\cdot 1 - (t+2)\cdot 1}{(t+1)^2}$$

I) $\dfrac{1}{1}$ $\quad$ II) $\dfrac{(t+1)\cdot 1 - (t+2)\cdot 1}{(t+1)^2}$

III) $\dfrac{(t+2)\cdot 1 - (t+1)\cdot 1}{(t+1)^2}$ $\quad$ IV) $\dfrac{(t+1)\cdot 1 + (t+2)\cdot 1}{(t+1)^2}$

e) $e(t) = \arctan(3t)$ $\quad e'(t) = \frac{1}{1+(3t)^2} \cdot D_t[3t] = \frac{1}{(1+3t^2)} \cdot 3$

I) $\dfrac{3}{1 + (3t)^2}$ $\quad$ II) $\dfrac{1}{1 + (3t)^2}$

III) $\dfrac{3}{\sec^2(3t)}$ $\quad$ IV) $\dfrac{3}{\sqrt{1 - (3t)^2}}$

5. (15 points) Find the derivatives with respect to x of the following functions. DO NOT SIMPLIFY YOUR DERIVATIVES.

$\boxed{D_x[\sec\theta] = \sec\theta\tan\theta}$

a) $a(x) = \sec(e^{(x^2+2)})$

$a'(x) = \sec(e^{(x^2+2)})\tan(e^{(x^2+2)}) \cdot D_x[e^{(x^2+2)}]$

$= \boxed{\sec(e^{(x^2+2)})\tan(e^{(x^2+2)}) \cdot e^{(x^2+2)} \cdot 2x}$

$a'(x) =$

b) $b(x) = 3^{(x^2)} \log_2(x)$

$b'(x) = (3^{(x^2)})' \log_2(x) + 3^{(x^2)}(\log_2(x))'$

$= \boxed{(3^{(x^2)} \cdot \ln 3 \cdot 2x)(\log_2(x)) + 3^{(x^2)} \cdot \frac{1}{x \cdot \ln 2}}$

$b'(x) =$

c) $c(x) = \sqrt{\ln(x)}$

$c'(x) = \boxed{\frac{1}{2\sqrt{\ln x}} \cdot D_x[\ln x] = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}}$

$c'(x) =$

6. (12 points) Hercules picks up a boulder and hurls it straight up. The height of the boulder in meters above the ground t seconds after Hercules releases the boulder is given by the function

$$h(t) = -5t^2 + 20t + 2.$$

a) Find a function for the velocity, $v(t)$, of the boulder t seconds after Hercules releases the boulder.

$v(t) = h'(t) = -10t + 20$

Answer: $v(t) = -10t + 20$

b) What is the velocity of the boulder at the instant that Hercules releases the boulder? Include units.

THE TIME OF "RELEASING THE BOULDER" IS AT TIME $t = 0$

$v(0) = -10(0) + 20 = 20$

Answer: 20 m/sec

c) At what time does the boulder reach its maximum height? Include units and **justify** your answer.

$v(t) = 0$ AT INSTANT OF MAX HEIGHT.

$-10t + 20 = 0$

$10t = 20$

$\boxed{t = 2 \text{ secs}}$

d) What is the maximum height attained by the boulder? Include units.

MAX HEIGHT IS ATTAINED AT $t = 2$ secs.

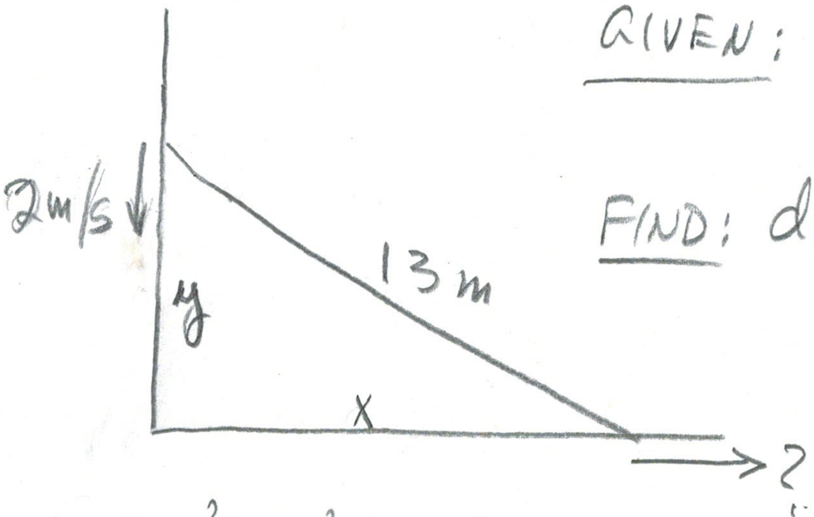
MAX HEIGHT AT $t = 2$ SECS IS:

$s(2) = -5(2^2) + 20(2) + 2$

Answer: $\boxed{-5(2^2) + 20(2) + 2 \text{ m}}$

$= -5(4) + 40 + 2 = 22 \text{ m}$

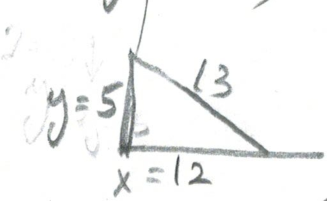
7. (10 points) Hercules is attempting to scale a Roman wall, so he leans a 13 meter ladder against the vertical wall. While he is climbing the ladder, the goddess Juno kicks the bottom of the ladder away from the wall causing the top of the ladder to start sliding down the wall. When the top of the ladder is 5 meters above the ground, the top of the ladder is sliding down the wall at a rate of 2 meters per second. At this same point in time, what is the speed of the bottom of the ladder as it moves away from the wall? Show work and include units.



GIVEN: $dy/dt = -2\text{ m/s}$ WHEN $y = 5\text{m}$

FIND: dx/dt WHEN $y = 5\text{m}$

WHEN $y = 5\text{m}$,



$x = 12\text{m}$

$$x^2 + y^2 = 13^2$$

$$D_t\left[x^2 + y^2\right] = D_t\left[13^2\right]$$

$$2x\,dx/dt + 2y\,dy/dt = 0$$

$$x\,dx/dt + y\,dy/dt = 0$$

$$x\,dx/dt = -y\,dy/dt$$

$$dx/dt = \frac{-y\,dy/dt}{x}$$

WHEN $y = 5\text{m}$,

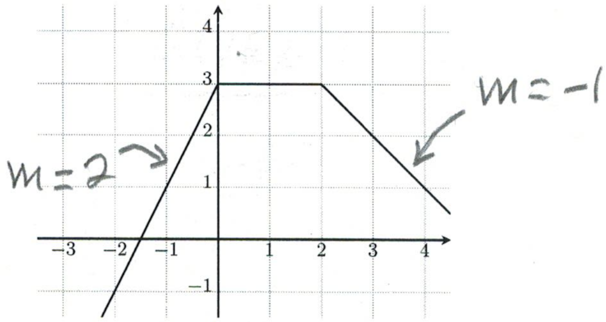
$$dx/dt = \frac{(-5\text{m})(-2\text{m/s})}{12\text{m}}$$

$$= \frac{5}{6}\text{ m/s}$$

Answer: $\frac{5}{6}$ m/s

8. (12 points) Multiple Choice: Consider the following two functions:

$f(x)$ is given by the following graph:



$g(x)$ is is a **differentiable** function, some of whose values are given in the following table

x	-2	-1	0	1	2	3
$g(x)$	4	-2	1	2	3	-4
$g'(x)$	-3	-1	0	3	2	5

Use the above functions to answer the following. Circle the correct answer. You do **not** need to show any work.

a) What is the derivative of $\dfrac{f(x)}{g(x)}$ at $x = 3$?

I) $-\frac{1}{5}$ II) $-\frac{3}{8}$ III) $\frac{3}{8}$ IV) $\frac{7}{8}$

$$D_x\left[\frac{f(x)}{g(x)}\right]\Bigg|_{x=3} = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \frac{(-4)(-1) - (2)(5)}{(-4)^2} = \frac{4-10}{16} = -\frac{6}{16} = -\frac{3}{8}$$

b) What is the derivative of $f(g(x))$ at $x = -2$?

I) 3 II) -6 III) -3 IV) 2

$$D_x[f(g(x))]\Big|_{x=-2} = f'(g(-2))\cdot g'(-2) = f'(4)\cdot(-3) = (-1)(-3) = 3$$

c) What is the derivative of $g(x)\cdot e^{(x^2)}$ at $x = 1$?

I) $7e$ II) $6e$ III) $-3e$ IV) $3e$

$$D_x\left[g(x)e^{(x^2)}\right]\Big|_{x=1} = \left[g'(x)e^{(x^2)} + g(x)\,e^{(x^2)}(2x)\right]\Big|_{x=1} = g'(1)e^{(1^2)} + g(1)\,e^{(1^2)}(2\cdot 1) = (3)e + (2)e(2) = 7e$$

$\boxed{\,' = \frac{d}{dx}}$

9. (5 points) Given the curve $e^y = 2x^3y^2 + e^2$, find dy/dx in terms of x and y. Show your work.

$$(e^y)' = (2x^3y^2 + e^2)'$$
$$e^y \cdot y' = 2\left[(x^3)'y^2 + x^3(y^2)'\right] + (e^2)'$$
$$e^y \cdot y' = 2\left[3x^2y^2 + x^3(2yy')\right] + 0$$
$$e^y \cdot y' = 6x^2y^2 + 4x^3yy'$$
$$e^y y' - 4x^3yy' = 6x^2y^2$$
$$y'(e^y - 4x^3y) = 6x^2y^2 \Rightarrow \boxed{y' = \frac{6x^2y^2}{e^y - 4x^3y}}$$

10. (5 points) Given the function $y = x^x$, find dy/dx in terms of x. Show your work.

$$D_x[x^x] = D_x[e^{x \cdot \ln x}] = e^{x \cdot \ln x} \cdot D_x[x \cdot \ln x]$$
$$= x^x \cdot [x' \cdot \ln x + x \cdot (\ln x)']$$
$$= x^x\left[\ln x + x \cdot \tfrac{1}{x}\right]$$
$$= \boxed{x^x[\ln x + 1]}$$

USING LOG DIFF:

$$y = x^x$$
$$\ln y = \ln(x^x)$$
$$\ln y = x \cdot \ln x$$
$$(\ln y)' = (x \cdot \ln x)'$$
$$\frac{1}{y} \cdot y' = x' \cdot \ln x + x \cdot (\ln x)'$$
$$y' = y\left[\ln x + x\left(\tfrac{1}{x}\right)\right] \Rightarrow \boxed{y' = x^x[\ln x + 1]}$$