1. A student throws a frisbee across Norlin quad. The function $s(t)$ gives the distance in yards the frisbee has flown after t seconds.

t in seconds	0	2	4	6	8	10	12	14	16
$s(t)$ in yards	0	15	28	39	48	55	60	63	64

(a) (3 points) What is the average velocity of the frisbee between $t = 2$ and $t = 10$ seconds? Include units.

$$\frac{s(10)-s(2)}{10-2} = \frac{55-15}{8} = \frac{40}{8} = \boxed{5 \text{ yards/sec}}$$

(b) (3 points) Estimate the instantaneous velocity at $t = 14$ seconds. Include units.

$$V_{\text{AVG on }[14,16]} = \frac{s(16)-s(14)}{16-14} = \frac{64-63}{2} = \frac{1}{2}$$

$$V_{\text{AVG on }[12,14]} = \frac{s(14)-s(12)}{14-12} = \frac{63-60}{2} = \frac{3}{2}$$

$$\text{INST VEL AT } t=14 \approx \frac{\frac{1}{2}+\frac{3}{2}}{2} = \frac{2}{2} = \boxed{1 \text{ yard/sec}}$$

(c) (3 points) Assume that $s'(8) = 4$. What does the value 4 represent in the context of the problem? Include units.

4 IS THE INSTANTANEOUS VELOCITY AT TIME 8 sec; 4 IS MEASURED IN UNITS OF yards/sec

2. Evaluate the following limits. Show your work.

(a) (4 points) $\lim_{x \to 0} \frac{e^{2x}}{\cos(2x)}$ $= \frac{e^{2 \cdot 0}}{\cos(2 \cdot 0)} = \frac{e^0}{\cos 0} = \frac{1}{1} = \boxed{1}$

(b) (4 points) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x}$

$= \lim_{x \to 2} \frac{(x-2)(x+1)}{x(x-2)}$

$= \lim_{x \to 2} \frac{x+1}{x}$

$= \frac{2+1}{2} = \boxed{\frac{3}{2}}$

3. Evaluate the following limits. Show your work.

(a) (6 points) $\lim_{x\to 1} \frac{2-\sqrt{3+x}}{x-1}$

$$= \lim_{x\to 1}\left[\frac{2-\sqrt{3+x}}{x-1}\cdot\frac{2+\sqrt{3+x}}{2+\sqrt{3+x}}\right]$$

$$= \lim_{x\to 1}\frac{4-(3+x)}{(x-1)(2+\sqrt{3+x})} = \lim_{x\to 1}\frac{1-x}{(x-1)(2+\sqrt{3+x})}$$

$$= \lim_{x\to 1}\frac{-1(x-1)}{(x-1)(2+\sqrt{3+x})} = \lim_{x\to 1}\frac{-1}{2+\sqrt{3+x}}$$

$$= \frac{-1}{2+\sqrt{3+1}} = \frac{-1}{2+2} = \boxed{-\frac{1}{4}}$$

(b) (6 points) $\lim_{x\to 3} \frac{|x-3|}{x^2-9}$

$$= \lim_{x\to 3}\frac{|x-3|}{(x-3)(x+3)} = \boxed{DNE}$$

BECAUSE LEFT AND RIGHT LIMITS ARE DIFFERENT

For $x>3$, $|x-3| = x-3$ so $\lim_{x\to 3+}\frac{|x-3|}{(x-3)(x+3)} = \lim_{x\to 3+}\frac{x-3}{(x-3)(x+3)}$

$$= \lim_{x\to 3+}\frac{1}{x+3} = \frac{1}{6}$$

For $x<3$, $|x-3| = -(x-3)$ so $\lim_{x\to 3-}\frac{|x-3|}{(x-3)(x+3)} = \lim_{x\to 3-}\frac{-(x-3)}{(x-3)(x+3)}$

$$= \lim_{x\to 3-}\frac{-1}{x+3} = -\frac{1}{6}$$

4. (3 points) Complete the definition of continuity.

A function f is continuous at a number a if:

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

5. (8 points) Consider the piece-wise function

$$f(x) = \begin{cases} e^{(bx-3)} & x < 1 \\ \ln(x) + 1 & x \geq 1. \end{cases}$$

Find the value of b that makes $f(x)$ continuous everywhere. Show your work.

e^{bx-3} IS CONT ~~EVERY~~ EVERYWHERE

$\ln(x) + 1$ IS CONT FOR $X \geq 1$

FIND b SO THAT $f(x)$ IS CONT at $a = 1$

$$\lim_{x \to 1} f(x) = f(1) = \ln(1) + 1 = 0 + 1 = 1$$

SO MUST HAVE $\displaystyle\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = 1$

$$1 = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} e^{bx-3} = e^{b \cdot 1 - 3} = e^{b-3}$$

$$e^{b-3} = 1 \Rightarrow b - 3 = 0 \Rightarrow \boxed{b = 3}$$

6. (10 points) Sketch the graph of a function $f(x)$ which satisfies ALL the conditions below. Draw any vertical and horizontal asymptotes.

(i) f has an infinite discontinuity at $x = -6$

(ii) $\lim_{x \to -3} f(x) = -\infty$

(iii) $\lim_{x \to 2^-} f(x) = 4$

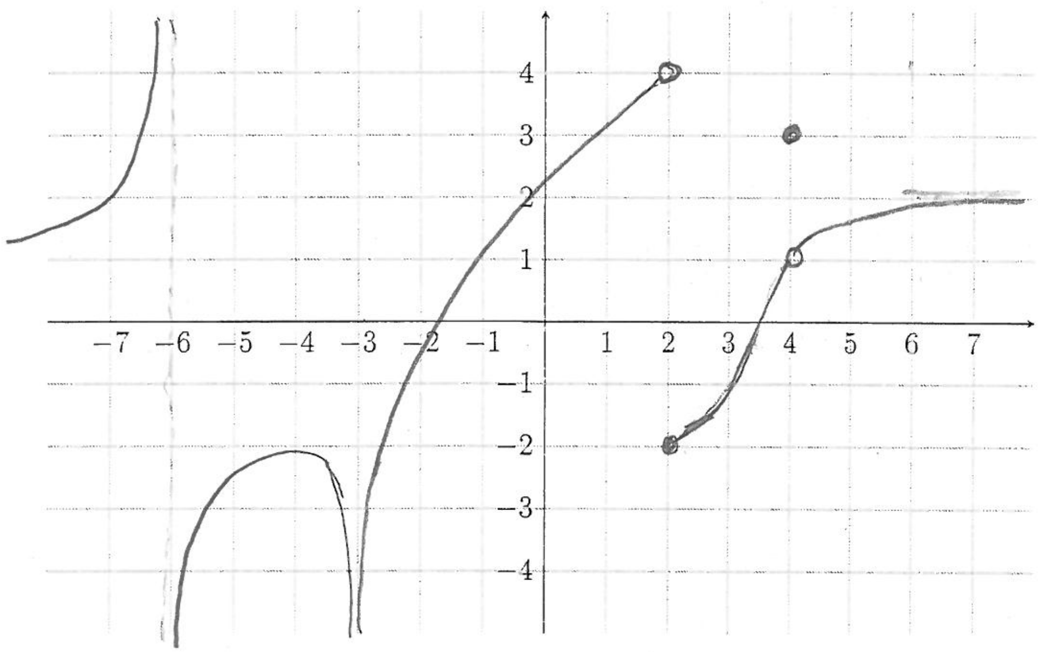
(iv) $\lim_{x \to 2^+} f(x) = -2$

(v) $f(2) = -2$

(vi) $\lim_{x \to 4} f(x) = 1$

(vii) f has a removable discontinuity at $x = 4$

(viii) $\lim_{x \to \infty} f(x) = 2$



7. (12 points) Multiple Choice. Evaluate the following limits. Circle the correct answer. You do **not** need to show work.

(a) $\displaystyle\lim_{t\to\infty} \frac{4t^2 - 3t + 2}{t^4 - 2t^2 + t - 5}$

I) 0   II) 4   III) ∞ IV) $-\infty$

deg NUM < deg DENOM

(b) $\displaystyle\lim_{x\to-\infty} \frac{6x^3 + x^2 - 4x + 1}{3x^3 - 2x^2 + 5}$

I) 0   II) 2   III) ∞ IV) $-\infty$

deg NUM = deg DENOM

RATIO OF LEADING COEFFS $= \frac{6}{3} = 2$

(c) $\displaystyle\lim_{x\to 1^+} 2^{(3/(x-1))}$

I) 0   II) 1   III) ∞ IV) $-\infty$

as $x \to 1^+$: $\frac{3}{x-1} \to \frac{3}{0^+} = +\infty$, THEN $2^{\frac{3}{x-1}} \to +\infty$ as $x \to 1^+$

(d) $\displaystyle\lim_{x\to 2} \frac{x^2 + 2x - 4}{x - 2}$

I) 0   II) $\frac{1}{2}$ III) 2   IV) DNE

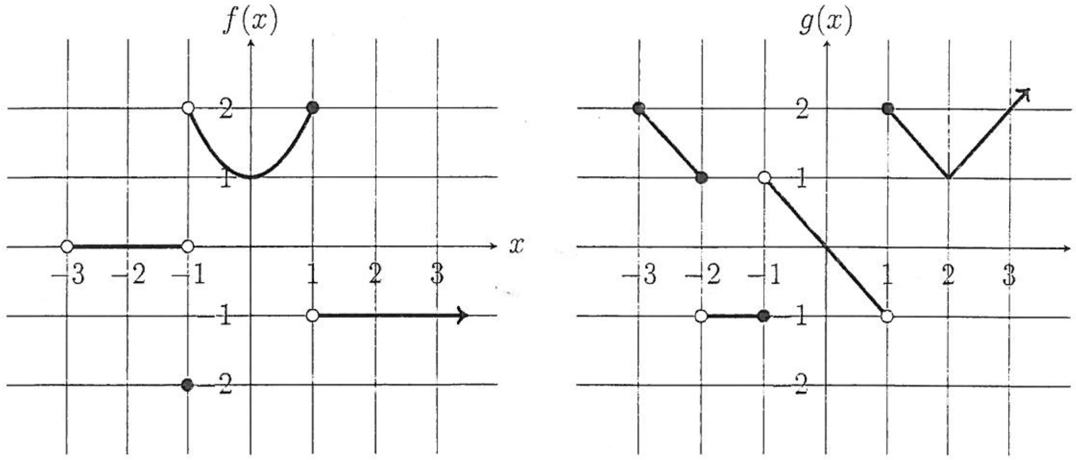
$x^2 + 2x - 4 \longrightarrow 2^2 + 2 \cdot 2 - 4 = 4$ as $x \to 2$

$x - 2 \longrightarrow 0^+$ if $x \to 2^+$

$x - 2 \longrightarrow 0^-$ if $x \to 2^-$

SO 1-SIDED LIMITS ARE DIFFERENT

8. (12 points) The graphs of two piece-wise functions, $f(x)$ and $g(x)$, are shown below.



Evaluate the following limits. Circle the correct answer. You do **not** need to show work.

(a) $\lim_{x\to 3} f(x)g(x)$ $= \left[\lim_{x\to 3} f(x)\right] \cdot \left[\lim_{x\to 3} g(x)\right] = (-1)\cdot(2) = -2$

I) 0   II) 1   III) -2   IV) DNE

(b) $\lim_{x\to 1} f(x) + g(x)$

$\lim_{x\to 1^+} (f(x)+g(x)) = -1 + 2 = 1$

SAME

I) 0   II) 1   III) 2   IV) DNE

$\lim_{x\to 1^-} (f(x)+g(x)) = 2 + (-1) = 1$

(c) $\lim_{x\to -2^-} \dfrac{f(x)}{g(x)}$ $f(x) \to 0$, $g(x) \to 1$ $= 0$

I) 0   II) -1   III) -2   IV) DNE

(d) $\lim_{x\to 2} f(g(x))$

I) 0   II) -1   III) 2   IV) DNE

$\lim_{x\to 2^+} f(g(x))$: $g(x) \to 1^+$ as $x \to 2^+$ and $f(x) \to -1$ as $x \to 1^+$

=

$\lim_{x\to 2^-} f(g(x))$: $g(x) \to 1^+$ as $x \to 2^-$ and $f(x) \to -1$ as $x \to 1^+$

9. (8 points) Evaluate the following limit. Show all of your work and be sure to cite any theorems you use.

$$\lim_{x\to 3}(x-3)^2\cos\left(\frac{1}{x-3}\right) = \boxed{0}$$

$$-1 \le \cos\left(\frac{1}{x-3}\right) \le 1 \quad \text{for } x \ne 3$$

$$-(x-3)^2 \le (x-3)^2 \cdot \cos\left(\frac{1}{x-3}\right) \le (x-3)^2$$

$-(x-3)^2 \to 0$ as $x \to 3$

$(x-3)^2 \to 0$ as $x \to 3$

so $(x-3)^2 \cdot \cos\left(\frac{1}{x-3}\right) \to \boxed{0}$ as $x \to 3$

BY SQUEEZE THM

10. (10 points) Use the limit definition of derivative to compute $f'(1)$ if $f(x) = x^2 + x$.

$$f(\sqcup) = \sqcup^2 + \sqcup$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

so $$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(1+h)^2 + 1+h\right] - \left[1^2+1\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} + 2h + h^2 + 1 + h - 2}{h}$$

$$= \lim_{h \to 0} \frac{3h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(3+h)}{h}$$

$$= \lim_{h \to 0} (3+h) = 3 + 0 = \boxed{3}$$

11. (8 points) Use the Intermediate Value Theorem to show that the equation

$$x^3 + x^2 + x - 2 = 0$$

has a solution in the interval $[0, 1]$. You must check that the hypotheses of the Intermediate Value Theorem are satisfied to receive full credit.

Let $f(x) = x^3 + x^2 + x - 2$

Then $f(0) = 0^3 + 0^2 + 0 - 2 = -2$

and $f(1) = 1^3 + 1^2 + 1 - 2 = 1$

OPPOSITE SIGNS, 0 IS IN BETWEEN

SO PICK $k = 0$

IVT: If f is CONT on $[0,1]$, then for every k between $f(0) = -2$ and $f(1) = 1$

[PICK $k = 0$]

THERE IS c in $[0,1]$ SUCH THAT $f(c) = 0$. c IS THE SOLUTION TO

$c^3 + c^2 + c - 2 = 0$

and $f(x)$ IS A POLYNOMIAL SO CONTINUOUS EVERYWHERE AND SO CONTINUOUS on $[0,1]$