1. Let $f(x) = \dfrac{12}{x}$.

(a) (10 points) Find $f'(2)$ using the **limit definition of derivative**.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{12}{2+h} - \frac{12}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{12\cdot 2 - 12(2+h)}{(2+h)(2)}}{h} = \lim_{h\to 0} \frac{-12\cancel{h}}{\cancel{(h)}(2+h)(2)}$$

$$= \lim_{h\to 0} \frac{-12}{(2+h)(2)} = \frac{-12}{(2+0)(2)} = -\frac{12}{4} = \boxed{-3}$$

(b) (4 points) Find an equation of the tangent line to $f(x)$ at $x = 2$.

$$y = f(2) = \frac{12}{2} = 6$$

LINE THRU $(x,y) = (2,6)$ WITH SLOPE $f'(2) = -3$

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 6 = -3(x-2)}$$

2. (8 points) Below is a partially completed proof of the fact that the function $f(x) = x^3 + x + 1$ has a zero (x-intercept) in the interval $[-1, 1]$. Complete the proof by filling in the blanks.

Proof:

$f(x)$ is CONTINUOUS on $[-1, 1]$ because $f(x)$ is a POLYNOMIAL

Since $f(-1) = -1$, and $f(1) = 3$, and 0 is between -1 and 3,

by the INTERMEDIATE VALUE Theorem

there exists an number c between $x = -1$ and $x = 1$ such that $f(c) = 0$

3. Let $f(x)$ be the following piecewise-defined function:

$$f(x) = \begin{cases} 2^x & \text{if } x < 1 \\ 3 + x & \text{if } x \geq 1 \end{cases}$$

(a) (4 points) Complete the definition of continuity at a point by filling in the blanks.

A function $f(x)$ is continuous at $x = a$ if $\boxed{\lim_{x \to a} f(x) = f(a)}$.

(b) (8 points) Is the function $f(x)$ continuous at $x = 1$? Why or why not? (Justify your answer using the definition of continuity.)

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3 + x) = 3 + 1 = 4$$

$\neq$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2^x = 2^1 = 2$$

So $\lim_{x \to 1} f(x)$ DOES NOT EXIST SINCE RT AND LEFT LIMITS ARE DIFFERENT

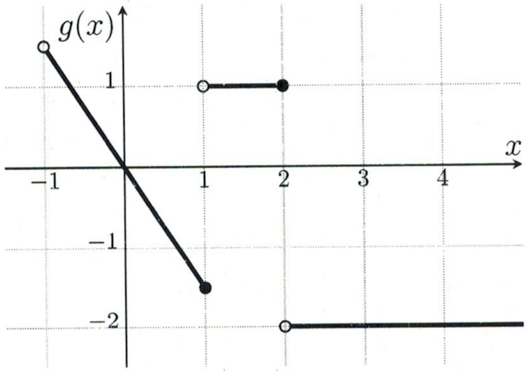
So $\underbrace{\lim_{x \to 1} f(x) \neq f(1)}_{\text{DONE}}$ So $\boxed{f \text{ IS } \underline{\text{NOT}} \text{ CONT AT } x=1}$

4. Consider the following three functions:

$f(x)$ is a **continuous** function, some of whose values are given in the following table:

x	-1	0	1	2	3
$f(x)$	0	1/2	2	3	7

$g(x)$ is given by the following graph:



$h(x)$ is given by the following piecewise function:

$$h(x) = \begin{cases} -2 & \text{if } x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$$

Evaluate the following limits, or write DNE if the limit doesn't exist. You **do not** need to show any work; you may write just an answer.

(a) (2 points) $\lim\limits_{x \to 2^-} (f(x) + g(x))^2$ $= (3+1)^2 = 4^2$ (a): 16

(b) (2 points) $\lim\limits_{x \to \infty} h(x)$ $= 1$ (b): 1

$\lim\limits_{x \to 2^+} [g(x) + h(x)] = -2 + 1$
$= -1$

(c) (2 points) $\lim\limits_{x \to 2}(g(x) + h(x))$ (c): -1

SAME

$\lim\limits_{x \to 2^-} [g(x) + h(x)] = 1 + (-2)$
$= -1$

(d) (2 points) $\lim\limits_{x \to 0}(f(x)h(x) + g(x))$ (d): -1

$= \left(\frac{1}{2}\right)(-2) + 0 = -1$

5. Evaluate the following limits. If there is no limit, then answer the best choice of ∞, $-\infty$ or DNE. Fully justify your answers using algebra, arithmetic or complete English sentences as appropriate.

(a) (4 points) $\lim_{x \to 2} \frac{x^2+3x-10}{x^2+4x-12}$ $= \lim_{x\to 2} \frac{(x+5)\cancel{(x-2)}}{(x+6)\cancel{(x-2)}}$

$= \lim_{x \to 2} \frac{x+5}{x+6} = \frac{2+5}{2+6} = \frac{7}{8}$

(a): $\boxed{7/8}$

(b) (4 points) $\lim_{x \to -1} \frac{x^2+5x+4}{5x^2-2x+1}$

$= \frac{(-1)^2+5(-1)+4}{5(-1)^2-2(-1)+1} = \frac{1-5+4}{5+2+1} = \frac{0}{8} = 0$

(b): $\boxed{0}$

(c) (4 points) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x \to 9} \frac{(x-9) \cdot 1}{(x-9)(\sqrt{x}+3)}$

$= \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$

(c): $\boxed{1/6}$

(d) (4 points) $\lim_{x \to 2^+} \frac{|2-x|}{2-x}$

FOR $x \to 2^+$, $x > 2$ SO $x-2>0$ SO $2-x<0$
SO $|2-x| = -(2-x)$

THEN $\lim_{x \to 2^+} \frac{|2-x|}{2-x} = \frac{-(2-x)}{2-x} = -1$

(d): $\boxed{-1}$

6. **Multiple Choice:** **Circle** the answer that represents the limit. If there is no limit, then circle the best choice of ∞, $-\infty$ or DNE (does not exist). You **do not** need to show any work.

(a) (2 points) $\displaystyle\lim_{x\to\infty} \frac{6x^3 - 9x^2 + 1}{-3x^2 + 2x - 8} =$

(I.) 2 (III.) ∞ (V.) 0

(II.) -2 (IV.) $-\infty$ (VI.) DNE

DEG (NUM) > DEG (DENOM)
SO LIMIT AT $\pm\infty$ IS $\pm\infty$

(b) (2 points) $\displaystyle\lim_{x\to-\infty} \frac{8x^3 + 3x^2 - 2x}{2x^3 - 7x + 1} =$

(I.) 4 (III.) ∞ (V.) 0

(II.) -4 (IV.) $-\infty$ (VI.) DNE

DEG(NUM) = DEG (DENOM)
SO LIMIT AT $\pm\infty$
IS RATIO OF
LEADING COEFFICIENTS
$\frac{8}{2} = 4$

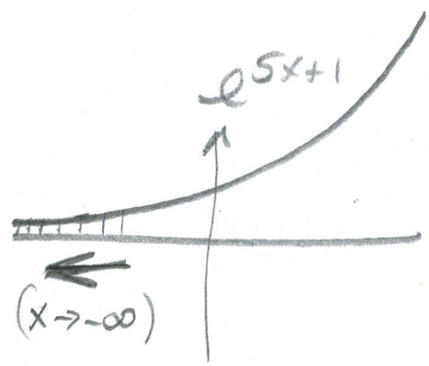
(c) (2 points) $\displaystyle\lim_{x\to3^+} \frac{5+x}{x-3} =$ $\quad 5+x \to +8, \quad x-3 \to 0^+$

" $\frac{8}{0^+} = +\infty$ "

(I.) 1 (III.) ∞ (V.) 0

(II.) -1 (IV.) $-\infty$ (VI.) DNE

(d) (2 points) $\displaystyle\lim_{x\to-\infty} e^{5x+1} =$



(I.) e (III.) ∞ (V.) 0

(II.) $-e$ (IV.) $-\infty$ (VI.) DNE

7. Krazy Kazoos, a company specializing in kazoos, wants to advertise its product in order to sell more units. Let $f(a)$ represent the number of kazoos sold when a dollars is spent on advertising. Through market research, they have found that

$$f(1000) = 800, \qquad f'(1000) = -3$$

(a) (4 points) Interpret $f(1000) = 800$ in the context of the problem, using complete English sentences. Include units.

WHEN \$1000 IS SPENT ON ADVERTISING, 800 KAZOOS ARE SOLD

(b) (4 points) Interpret $f'(1000) = -3$ in the context of the problem, using complete English sentences. Include units.

WHEN \$1000 IS SPENT ON ADVERTISING,

3 FEWER KAZOOS ARE BEING SOLD PER DOLLAR

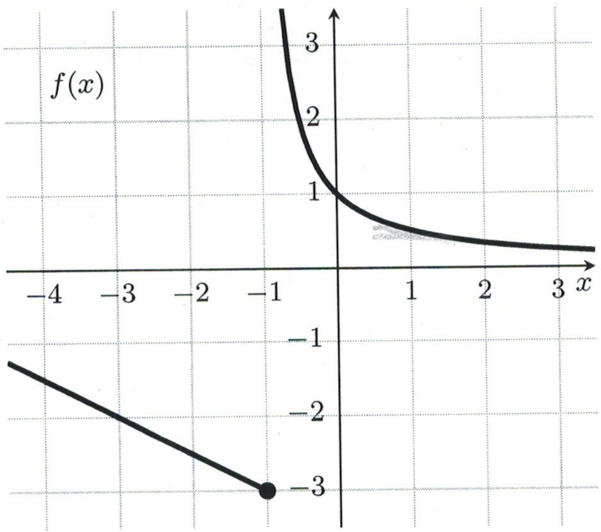
OR

KAZOOS ARE BEING SOLD AT THE RATE OF -3 KAZOOS/\$

OR

IF ANOTHER DOLLAR IS SPENT ON ADVERTISING, THEN APPROXIMATELY $800 - 3 = 797$ KAZOOS WILL BE SOLD

8. **Multiple Choice: Circle the correct answer for each question.** The graph of function $f(x)$ is shown below.



(a) (2 points) What is the value of $f'(-3)$?

(I.) -2 (II.) 2 (III.) $-\frac{1}{2}$ (IV.) $\frac{1}{2}$

SLOPE OF ENTIRE LINE SEGMENT : $\Delta y / \Delta x = \frac{-1}{2}$

(b) (2 points) Is the value of $f'(x)$ positive, negative, zero, or undefined at $x = 1$?

(I.) positive (II.) negative (III.) zero (IV.) undefined (DNE)

TAN LINE AT X=1 SLOPES DOWN AND TO THE RIGHT ; NEGATIVE SLOPE

(c) (2 points) Is the value of $f'(x)$ positive, negative, zero, undefined at $x = -1$?

(I.) positive (II.) negative (III.) zero (IV.) undefined (DNE)

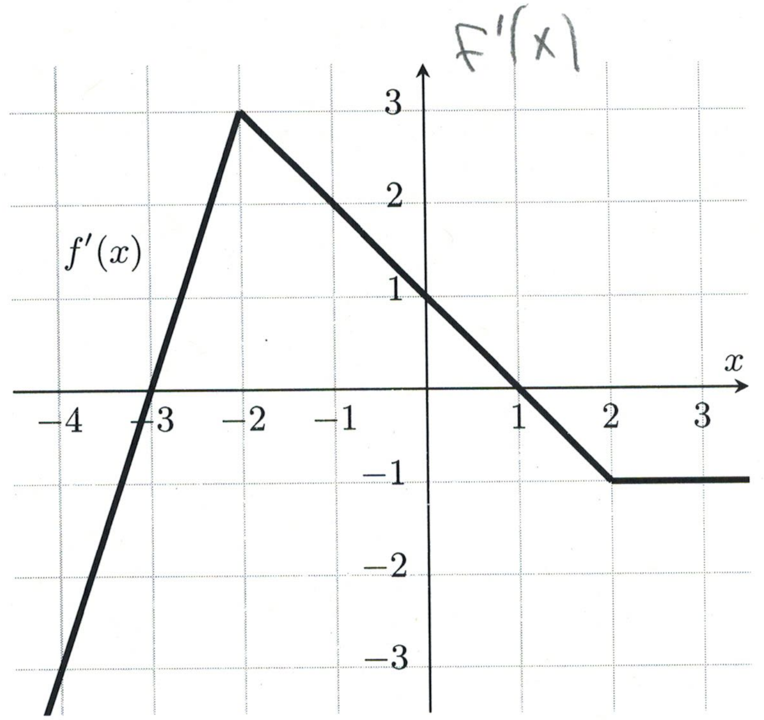
f NOT CONT AT X = −1

(d) (2 points) Is the value of $f''(x)$ positive, negative, zero, or undefined at $x = 1$?

(I.) positive (II.) negative (III.) zero (IV.) undefined (DNE)

f is CU at x = 1

9. The graph of $f'(x)$ (the **derivative** of $f(x)$) is shown below.



Graph of $f'(x)$

(a) (4 points) On what interval(s) is $f(x)$ increasing?

f(x) INCR WHERE f'(x) > 0

(a): $(-3, 1)$

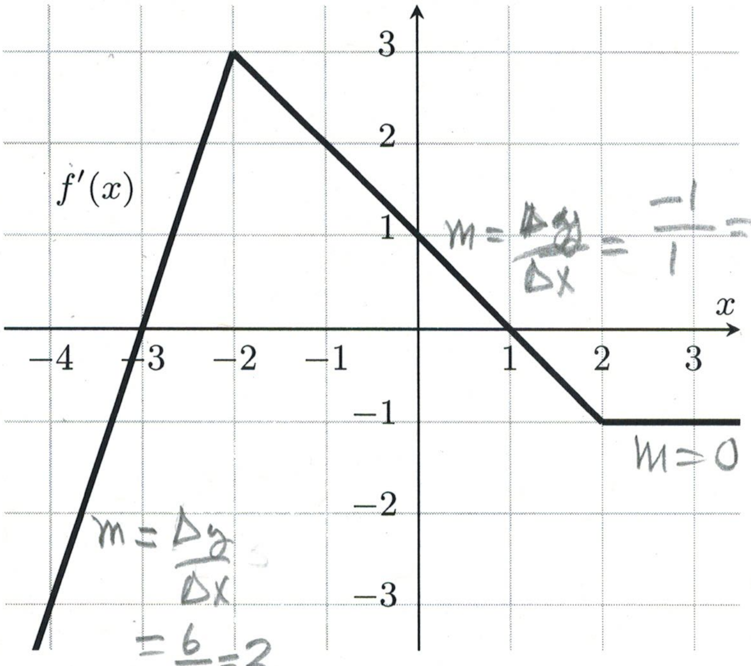
(b) (4 points) On what interval(s) is $f(x)$ concave down?

f(x) IS CD WHERE

f'(x) IS DECR

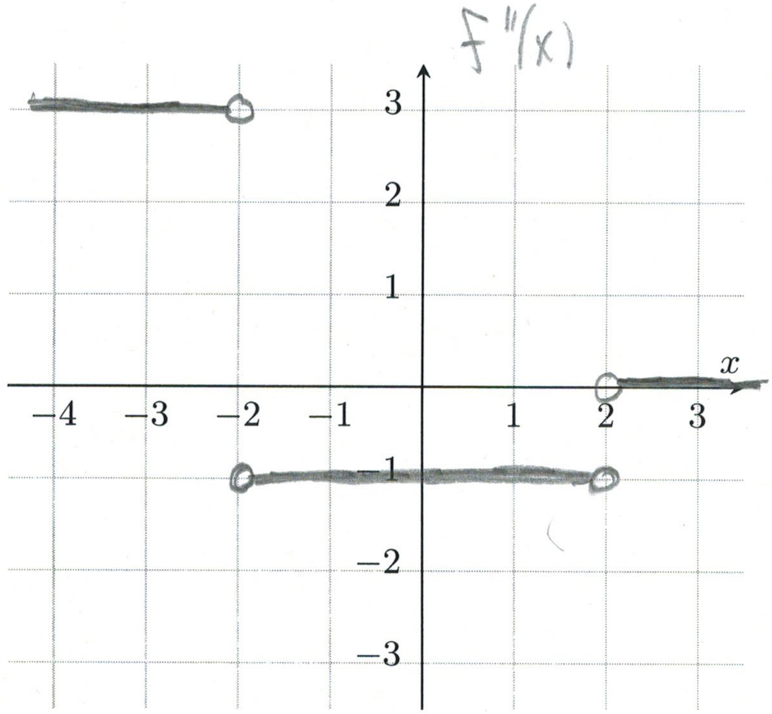
(b): $(-2, 2)$

10. (10 points) The graph of $f'(x)$ is reproduced below. It is the same as the graph of $f'(x)$ on the previous page. Use the blank coordinate plane below to sketch a graph of $f''(x)$.



Graph of $f'(x)$

$f''(-2) = \text{DNE}$ SINCE $f'(x)$ HAS A CORNER AT $x = -2$



$f''(2) = \text{DNE}$ SINCE $f'(x)$ HAS A CORNER AT $x = 2$

Graph of $f''(x)$