1. For each of the following statements, determine if they are true or false. If true, show or explain why it is true. If false, explain why or give a counterexample.

(a) If $f''(c) = 0$, then $f(x)$ has an inflection point at $x = c$.

FALSE, RECALL THAT AN INFLECTION POINT IS DEFINED AS A POINT AT WHICH A FUNCTION CHANGES CONCAVITY. CONSIDER $f(x)=2$, THEN $f'(x)=0$, AND $f''(x)=0$. OBSERVE THE SECOND DERIVATIVE IS ZERO FOR ALL X, BUT THE ORIGINAL FUNCTION IS A HORIZONTAL LINE AT $y=2$ THAT HAS NO CONCAVITY BEHAVIOR.

(b) The line tangent to $f(x)$ at $x = a$ will only intersect the graph of $f(x)$ at one point.

FALSE, CONSIDER $f(x)=\cos(x)$ AT $x=0$. THIS HAS THE TANGENT LINE $y=1$. WHEN WE LOOK AT THE GRAPH WE SEE THAT THE TANGENT LINE ACTUALLY INTERSECTS $f(x)$ AN INFINITE NUMBER OF TIMES.

(c) If $f(x)$ is a differentiable function, then $f(x)$ is a continuous function.

TRUE, THE DERIVATIVE OF $f(x)$ AT $X=a$ IS DEFINED AS $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ OR $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$.

ASSUME $f(x)$ IS DIFFERENTIABLE AT $x=a$, THEN WE CAN WRITE

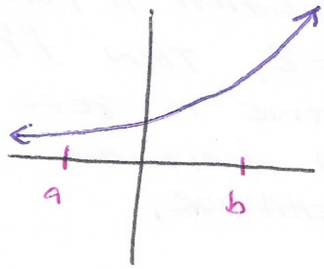
$$\lim_{x\to a}(f(x)-f(a))=\left(\lim_{x\to a}(x-a)\right)\underbrace{\left(\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\right)}_{f'(a)}=\underbrace{\lim_{x\to a}(x-a)}_{0}\cdot f'(a)=0\cdot f'(a)=0$$

THEN $\lim_{x\to a} f(x)-f(a)=0$, WHICH IMPLIES $\lim_{x\to a} f(x)=f(a)$.

THUS, $f(x)$ IS CONTINUOUS AT $X=a$.

(d) If $f''(x) > 0$ on the interval (a, b), then $f'(x) < 0$ on the interval (a, b).

FALSE, IF $f''(x) > 0$ ON THE INTERVAL (a,b), THEN $f(x)$ IS CONCAVE UP ON THE INTERVAL (a,b). THIS DOES NOT FORCE $f'(x) < 0$ (I.E. THE FUNCTION TO BE DECREASING) ON THE INTERVAL (a,b). CONSIDER THE FOLLOWING COUNTEREXAMPLE:



$f(x) = e^x$

THIS FUNCTION IS CONCAVE UP EVERYWHERE AND INCREASING EVERYWHERE.

(e) If $f(x)$ is a polynomial, then it is differentiable for all x.

TRUE, RECALL A POLYNOMIAL FUNCTION IS OF THE FORM

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$ WHERE n IS A NATURAL NUMBER AND a_n IS A CONSTANT FOR ALL n.

WE CAN DIFFERENTIATE EACH TERM OF THIS POLYNOMIAL USING THE POWER RULE AND KNOWING THE DERIVATIVE OF A CONSTANT IS ZERO. WE CAN PUT ALL THESE DERIVATIVES TOGETHER FOLLOWING ADDITION DIFFERENTIATION RULE TO GET

$$f'(x) = n(a_n)x^{n-1} + (n-1)(a_{n-1})x^{n-2} + \cdots + a_1$$

WHICH EXISTS FOR ALL VALUES OF X.

(f) If g is differentiable at $x = a$ and f is differentiable at $x = g(a)$, then $f \circ g$ is differentiable at $x = a$.

TRUE, THIS IS SIMPLY THE CHAIN RULE.

LET $h(x) = f(g(x))$, THEN $h'(x) = f'(g(x))g'(x)$.

THEN LOOKING AT THIS WHEN $x=a$, WE HAVE

$$h'(a) = f'(g(a))g'(a)$$

SINCE g IS DIFFERENTIABLE AT $x=a$, $g'(a)$ EXISTS.

SINCE f IS DIFFERENTIABLE AT $x=g(a)$, $f'(g(a))$ EXISTS.

THEREFORE, $h'(a) = f'(g(a))g'(a)$ EXISTS WHICH IMPLIES $f \circ g$ IS DIFFERENTIABLE AT $x=a$.