1. For each of the following statements, determine if they are true or false. If true, show or explain why it is true. If false, explain why or give a counterexample.

(a) If $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist.

FALSE, CONSIDER THE CASE WHEN $f(x) = x^2 - 4$, $g(x) = x+2$ WHEN WE LET $x \to -2$.

$\lim_{x\to -2} x+2 = 0$, BUT $\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} x-2 = -4$.

THIS IS AN EXAMPLE OF A HOLE OR REMOVABLE DISCONTINUITY.

(b) If $\lim_{x\to a} g(x) = \infty$ and $\lim_{x\to a} f(x)$ exists then $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$

TRUE, IF $\lim_{x\to 9} f(x)$ EXISTS AS SOME CONSTANT VALUE AND $\lim_{x\to 9} g(x) = \infty$, THEN THE DENOMINATOR IS GROWING AT A FASTER RATE THAN NUMERATOR. DIVIDING BY A LARGER AND LARGER NUMBER RESULTS IN THE LIMIT VALUE APPROACHING ZERO.

(c) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a}(f(x) + g(x))$ exists then $\lim_{x\to a} g(x)$ exists.

TRUE,

FROM LIMIT LAWS, WE GET THAT $\lim_{x\to 9} \left((f(x)+g(x)) - f(x) \right)$ MUST EXIST SINCE $\lim_{x\to 9} f(x)$ AND $\lim_{x\to 9} (f(x)+g(x))$ EXIST.

THEREFORE,

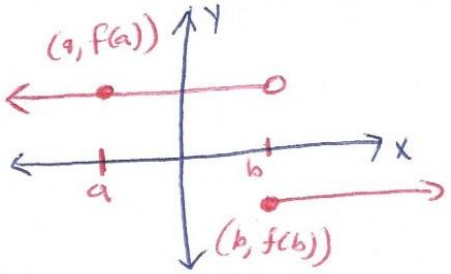
$$\lim_{x\to 9} \left((f(x)+g(x)) - f(x) \right) = \lim_{x\to 9} g(x)$$

SO $\lim_{x\to 9} g(x)$ EXISTS.

(d) If f is a function with $f(a) > 0$ and $f(b) < 0$, then $f(x) = 0$ for some $a < x < b$.

FALSE,

WE COULD HAVE A JUMP DISCONTINUITY.
CONSIDER THE COUNTEREXAMPLE:



HERE $f(a) > 0$ AND $f(b) \leq 0$, BUT THERE IS NO x BETWEEN a AND b SUCH THAT $f(x) = 0$.

NOTE THAT IF f WAS A CONTINUOUS FUNCTION THIS WOULD BE TRUE BY INTERMEDIATE VALUE THEOREM.

(e) If $f(x) \leq g(x) \leq h(x)$ for all x with $\lim_{x \to a} f(x) = -1$ and $\lim_{x \to a} h(x) = 1$, then $\lim_{x \to a} g(x)$ exists with $-1 \leq \lim_{x \to a} g(x) \leq 1$.

FALSE,

THIS IS ALSO NOT TRUE SINCE WE DO NOT REQUIRE f, g, h TO BE CONTINUOUS. CONSIDER THE COUNTEREXAMPLE: