Math 1300, Final Exam

May 4, 2016

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Cherry Ng	8:00 - 8:50
☐	Section 002	Albert Bronstein	9:00 - 9:50
☐	Section 003	Katharine Adamyk	9:00 - 9:50
☐	Section 004	Sebastian Bozlee	10:00 - 10:50
☐	Section 005	Hanson Smith	10:00 - 10:50
☐	Section 006	Braden Balentine	10:00 - 10:50
☐	Section 007	Matthew Pierson	11:00 - 11:50
☐	Section 008	Ira Becker	11:00 - 11:50
☐	Section 009	Charlie Scherer	12:00 - 12:50
☐	Section 010	Tyler Schrock	12:00 - 12:50
☐	Section 011	Albert Bronstein	1:00 - 1:50
☐	Section 012	Sarah Salmon	1:00 - 1:50
☐	Section 013	Athena Sparks	2:00 - 2:50
☐	Section 014	Leo Herr	2:00 - 2:50
☐	Section 015	Ira Becker	3:00 - 3:50
☐	Section 016	Jun Hong	3:00 - 3:50
☐	Section 017	Ilia Mishev	4:00 - 4:50
☐	Section 430R	Patrick Newberry	11:00 - 11:50

Question	Points	Score
1	12	
2	5	
3	6	
4	6	
5	5	
6	12	
7	6	
8	7	
9	6	
10	9	
11	9	
12	10	
13	7	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 2.5 hours and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) Evaluate the following derivatives. You do not have to show your work and should NOT simplify after differentiating.

(a) $\frac{d}{dx}\left[x^3 + 5x - \frac{12}{x}\right]$

$$3x^2 + 5 + \frac{12}{x^2}$$

(b) $\frac{d}{dx}\left[\arcsin(x) + 2^x + 2\right]$

$$\frac{1}{\sqrt{1-x^2}} + 2^x \cdot \ln 2$$

THIS IS OK

(c) $\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)+1}\right]$

$$\frac{(\cos x+1)\cos x - \sin x(-\sin x)}{(\cos x+1)^2}$$

$$\frac{\cos^2 x + \cos x + \sin^2 x}{(\cos x+1)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

(d) $\frac{d}{dx}\left[\int_3^{e^x} \ln(t)\, dt\right] = \ln(e^x)e^x = x \cdot e^x$

2. (5 points) Use the Intermediate Value Theorem to prove that the function

$$f(x) = \cos(x) - x$$

has a zero on the interval $[0, \pi]$. You must check that the hypotheses of the Intermediate Value Theorem are satisfied to receive full credit.

Both x and $\cos x$ are continuous and so is $f(x) = \cos x - x$. IVT Applies to $f(x) = \cos x - x$

$f(0) = \cos(0) - 0 = 1 > 0$

$f(\pi) = \cos(\pi) - \pi) = -1 - \pi < 0$

By IVT THERE is A c with $0 < c < \pi$ such that $f(c) = 0$ OR $\cos(c) - c = 0$

so $f(x) = \cos x - x$ has a ZERO in the interval $[0, \pi]$.

3. (6 points) Consider the two curves $y = 6(1 + x - x^2)$ and $y = 6(3 - 2x)$.

(a) At what x-values do these two curves intersect? Show all work that leads to your answer.

$6(1+x-x^2) = 6(3-2x)$ Divide by 6

$1+x-x^2 = 3-2x$

$0 = x^2-3x+2$

$0 = (x-1)(x-2)$

$x = 1, 2$

(b) Calculate the area bounded between these two curves. Show all work that leads to your answer.

$1 < \frac{3}{2} < 2 \quad y(\frac{3}{2}) = 6(1+\frac{3}{2}-\frac{9}{4}) = 6\left[\frac{4+6-9}{4}\right] = \frac{6}{4} = \frac{3}{2}$

$y(\frac{3}{2}) = 6(3-2\cdot\frac{3}{2}) = 0$

$y=6(1+x-x^2)$ is Above $y=6(3-2x)$ on the interval $[1,2]$

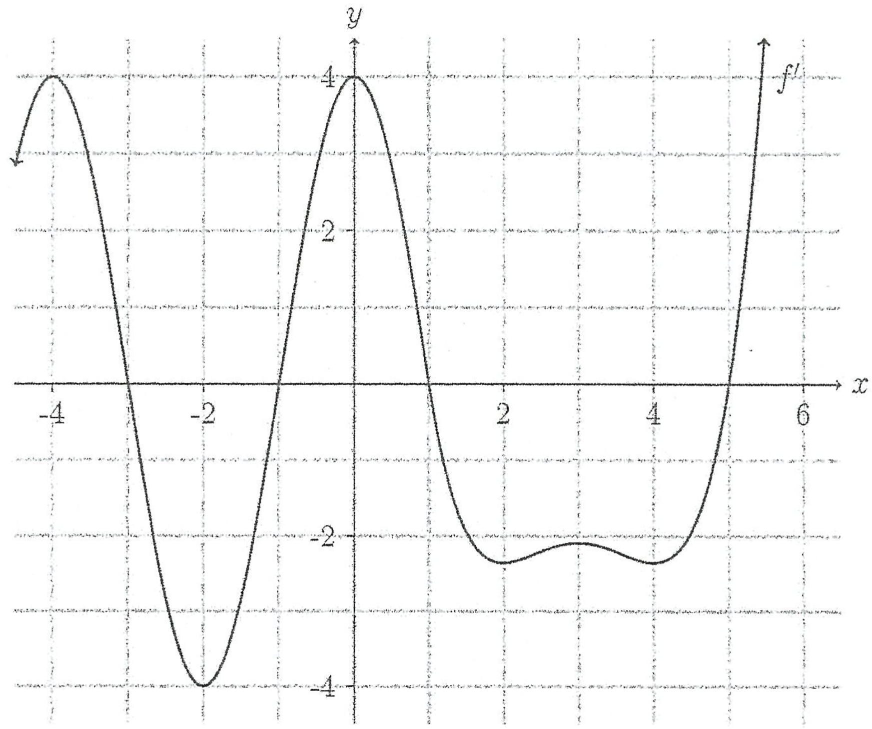
$$\text{AREA} = \int_1^2 \left[6(1+x-x^2) - 6(3-2x)\right]dx =$$

$$\int_1^2 (6+6x-6x^2-18+12x)dx = \int_1^2(-6x^2+18x-12)dx =$$

$$-2x^3+9x^2-12x\Big|_1^2 = (-16+36-24)-(-2+9-12) =$$

$$-4+5=1$$

4. (6 points) Given below is the graph of f', the **derivative** of f. (You do not need to show work for the following questions.)



(a) On what open interval(s) is f decreasing?

A. $(-\infty, -3) \cup (-1, 1) \cup (5, \infty)$

B. $(-1, 1) \cup (2, 4)$

C. $\boxed{(-3, -1) \cup (1, 5)}$ f is decreasing when $f' < 0$

D. $(-4, -2) \cup (0, 2) \cup (3, 4)$

(b) On what open interval(s) is f' decreasing?

A. $\boxed{(-4, -2) \cup (0, 2) \cup (3, 4)}$

B. $(-3, -1) \cup (1, 5)$

C. $(-\infty, -4) \cup (-2, 0) \cup (2, 3) \cup (4, \infty)$

D. $(-4, -2) \cup (0, 2)$

(c) On what open interval(s) is f'' decreasing?

f'' is decreasing when $(f'')'$ or $f''' < 0$ but f''' is the 2nd derivative of f' so f'' is decreasing when f' is concave down

A. $(-\infty, -3) \cup (5, \infty)$

B. $\boxed{(-\infty, -3) \cup (-1, 1) \cup (2.5, 3.5)}$

C. $(-3, -1) \cup (1, 2.5) \cup (3.5, \infty)$

D. $(-\infty, -3) \cup (-2, 2) \cup (2, 4)$

5. (5 points) A beekeeper begins maintaining an increasing population of honeybees. Let $n(t)$ represent the number of honeybees in the population t weeks after the beekeeper begins maintenance.

(a) What does

$$\int_0^{20} n'(t)\,dt$$

represent? Include appropriate units.

$\int_0^{20} n'(t)dt = n(t)\Big|_0^{20} = n(20)-n(0)$ honeybees

The integral is the net change in the number of honeybees from the start of week 1 to the end of week 20.

(b) Suppose

$$n(t) = 200e^{0.05t}.$$

Use this function to find $\int_0^{20} n'(t)\,dt$. Show all work and do not simplify your answer. Include units.

$\int_0^{20} n'(t)dt = n(t)\Big|_0^{20} = 200e^{0.05t}\Big|_0^{20} =$

$200[e-1]$ honeybees

6. (12 points) Evaluate the following and please show your work. You do not need to simplify numerical expressions.

(a) $\int \left(9x^2 - 5 + \frac{3}{x}\right)\, dx$

$\frac{9}{3}x^3 - 5x + 3\ln|x| + C$

$3x^3 - 5x + \ln|x|^3 + C$

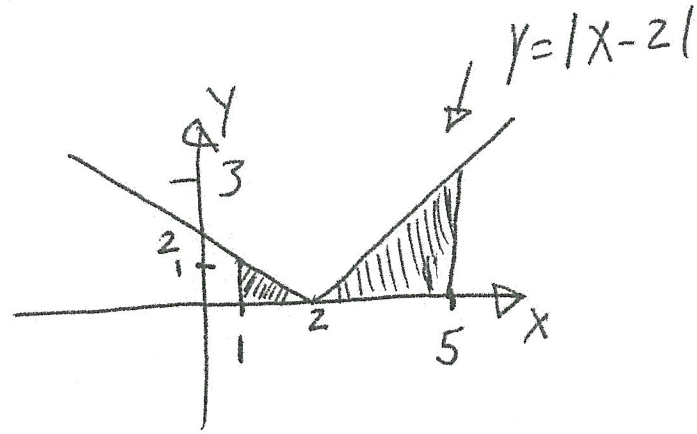
(b) $\int_0^1 \frac{xe^x - x^3}{x}\, dx = \int_0^1 (e^x - x^2)\,dx = e^x - \frac{1}{3}x^3 \Big|_0^1 =$

$(e - \frac{1}{3}) - (1 - 0) = e - 4/3$

(c) $\int \frac{\ln x}{x}\, dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx$

$\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$

(d) $\int_1^5 |x-2|\, dx =$



$\frac{1}{2}1\cdot 1 + \frac{1}{2}3\cdot 3 =$

$\frac{1}{2} + \frac{9}{2} = \frac{10}{2} = 5$

7. (6 points) Find all locations (x-values) of inflection points of $h(x) = e^{-2x^2}$. Fully justify your answer.

$$h'(x) = -4xe^{-2x^2}$$

$$h''(x) = -4x \cdot (-4xe^{-2x^2}) + e^{-2x^2} \cdot (-4) =$$

$$16x^2 e^{-2x^2} - 4e^{-2x^2} = 4e^{-2x^2}[4x^2-1]$$

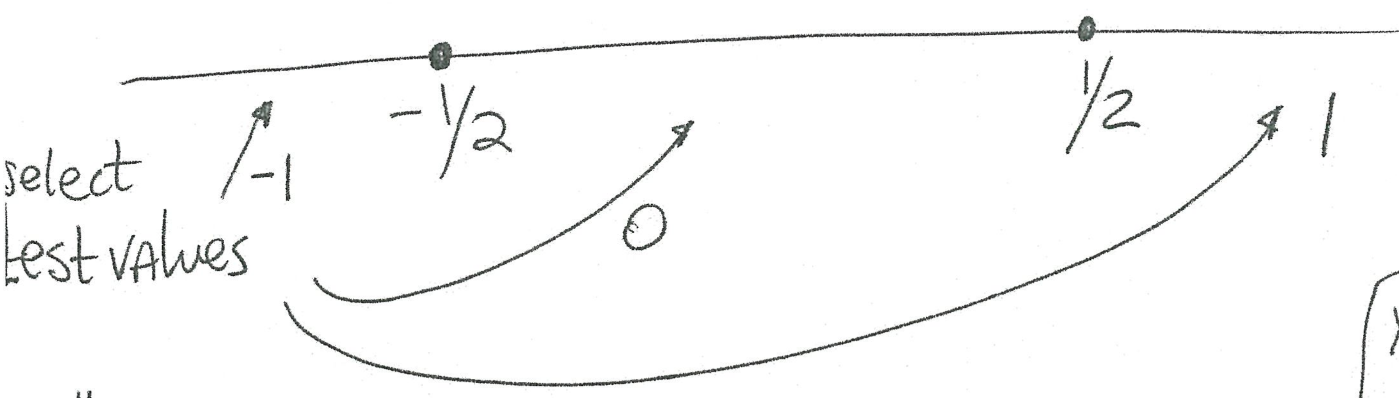
$h''(x)$ is Always defined (defined for All x)

$h''(x) = 0 \quad 4e^{-2x^2}[4x^2-1] = 0 \quad 4x^2-1=0$

never $=0$ and Always >0

$x^2 = 1/4$

$x = \pm 1/2$



$h''(-1) = 4e^{-2} \cdot 3 > 0$ CU on $(-\infty, -1/2)$

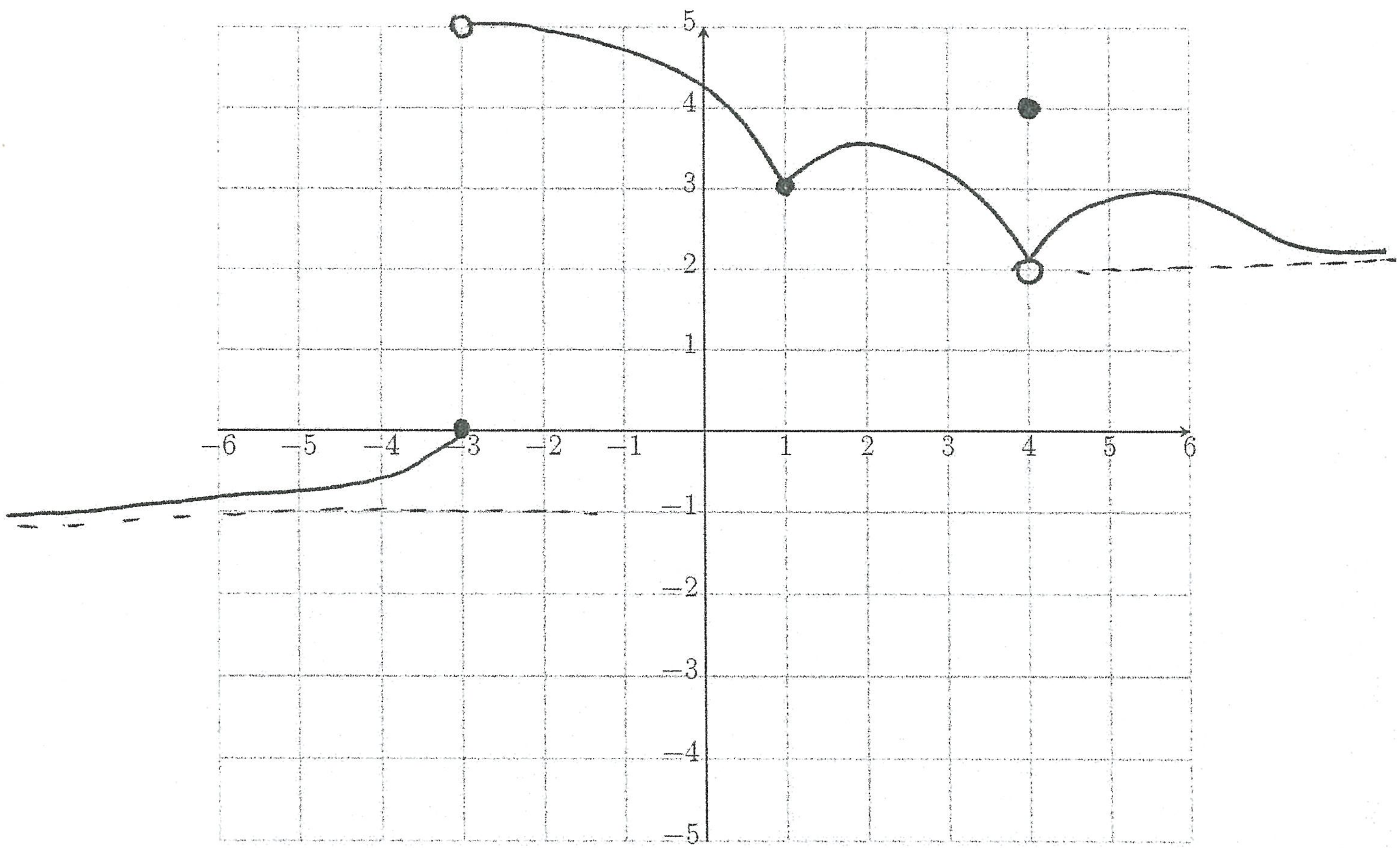
$h''(0) = 4(-1) < 0$ CD on $(-1/2, 1/2)$

$h''(1) = 4e^{-2} \cdot 3 > 0$ CU on $(1/2, \infty)$

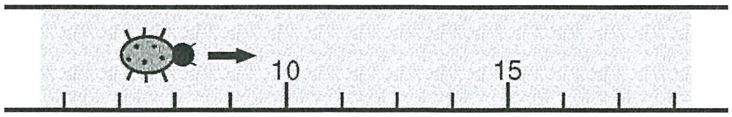
$x = -\frac{1}{2}$ and $x = \frac{1}{2}$ are inflection points (x-coordinates) of h

8. (7 points) On the axes below, draw the graph of a function $f(x)$ which has all of the following properties:

- f is defined on $(-\infty, \infty)$
- $f(1) = 3$
- f is not continuous at $x = -3$ and $\lim_{x \to -3} f(x)$ does not exist
- f is not continuous at $x = 4$ but $\lim_{x \to 4} f(x) = 2$
- f is continuous but not differentiable at $x = 1$
- $\lim_{x \to \infty} f(x) = 2$
- $\lim_{x \to -\infty} f(x) = -1$



9. (6 points) A bug is placed at the 7 cm mark on a meter stick and begins walking along the meter stick.



It always walks in the positive direction (rightwards). Its velocities at certain instants while it is on the meter stick are given below:

time since the bug was placed on the meter stick (seconds)	0	2	4	6	8	10
velocity of the bug at that time (centimeters/second)	1	1	2	2	1	3

Assume the velocity, $v(t)$, of the bug is a continuous function.

(a) Using a right Riemann sum with n=5 rectangles, estimate $\int_0^{10} v(t)\,dt$. Show all work and include units.

with $n=5$ $\Delta x = \frac{10-0}{5} = 2$

$\int_0^{10} v(t)dt \approx 2[1+2+2+1+3] = 18$ cm

(b) At approximately what centimeter mark is the bug ten seconds after being placed on the meter stick? Show all work and include units.

7cm + 18cm = 25 cm mark

(c) What was the average acceleration of the bug during this ten-second time interval? Show all work and include units.

Ave Acceleration$_{[0,10]}$ $= \frac{V(10)-V(0)}{10-0} = \frac{3-1}{10} =$

$1/5$ cm/sec^2

10. (9 points) If g is an integrable function which satisfies

$$\int_2^6 g(x)\,dx = 9 \text{ and } \int_2^3 g(x)\,dx = 11,$$

choose the correct value for each of the following definite integrals. You do not need to show work.

(a) $\int_3^6 g(x)\,dx$

$$\int_2^6 g(x)dx = \int_2^3 g(x)dx + \int_3^6 g(x)dx$$

$$9 = 11 + \int_3^6 g(x)dx$$

$$\int_3^6 g(x)dx = -2$$

A. 4

B. -1

C. 2

$\boxed{\text{D. -2}}$

(b) $\int_2^6 (4g(x) - 2)\,dx$ $= 4\int_2^6 g(x)dx - 2\int_2^6 1\,dx =$

$$4 \cdot 9 - 2x\Big|_2^6 = 36 - 8 = 28$$

A. 20

$\boxed{\text{B. 28}}$

C. 5

D. 36

(c) $\int_{\frac{2}{3}}^2 g(3x)\,dx$

$u = 3x$ $\quad x = \frac{2}{3}$ $\quad u = 2$

$du = 3dx$ $\quad x = 2$ $\quad u = 6$

$\frac{1}{3}du = dx$

$$\frac{1}{3}\int_2^6 g(u)du = \frac{1}{3} \cdot 9 = 3$$

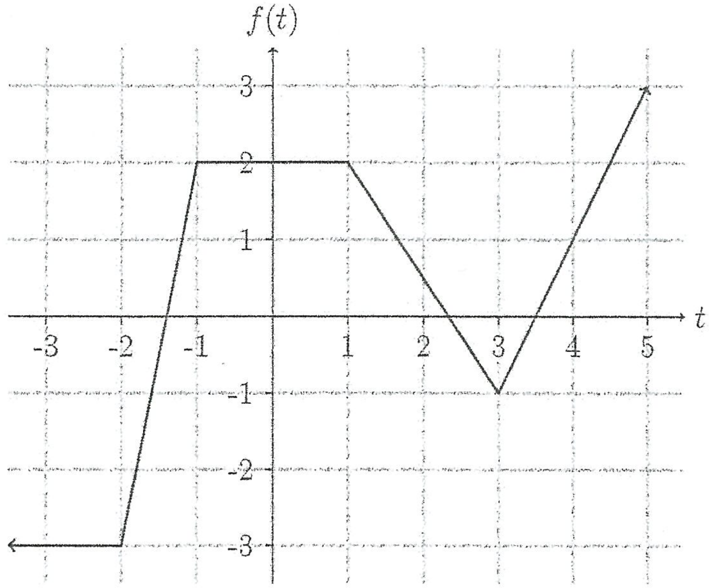
A. $-2/3$

$\boxed{\text{B. 3}}$

C. 11/3

D. 27

11. (9 points) Given below is the graph of $f(t)$.



Define the function $F(x)$ by

$$F(x) = \int_{-1}^{x} f(t)\, dt.$$

(a) Find the following:

$F(-1) =$ 0 $\qquad\qquad$ $F(1) =$ 4

$F'(1) =$ 2 $\qquad\qquad$ $F''(4) =$ 2

Slope of the line through points (3, -1) And (4, 1)

(b) List the x-value(s) of the local minima of $F(x)$: $x = -1.5,\ 3.5$

(c) Find a formula for $F(x)$ between $x = 0$ and $x = 1$. Show your work.

Let $\quad 0 \leq x \leq 1 \quad F(x) = \int_{-1}^{x} f(t)\,dt = \int_{-1}^{0} f(t)\,dt + \int_{0}^{x} f(t)\,dt =$

$2 + 2x$

$F'(x) = f(x) = 0$ when $x = -3/2,\ 2.\overset{\approx}{3},$ and 3.5

$- - - - \ + + + + + + \ - - - - \ + + + +$ $\quad F'(x) = f(x)$

$-1.5 \qquad 2.3 \qquad 3.5$

12. (10 points) Evaluate the following limits and choose the best answer. You do not need to show your work.

(a) $\lim_{x\to\infty} \frac{9x^2-3}{4x^2+3}$

$\boxed{\text{A. } 9/4}$

B. 0

C. -1

D. Does not exist

(b) $\lim_{x\to 0^-} \frac{2}{x^3}$

$2 / \text{small } (-) \text{ numbers} \longrightarrow -\infty$

A. 0

B. ∞

$\boxed{\text{C. } -\infty}$

D. Does not exist

(c) $\lim_{x\to\infty} \frac{2\ln(x)}{\ln(x)+1}$

$\frac{\infty}{\infty}$ L'H

$\lim_{x\to\infty} \frac{2/x}{1/x} = \lim_{x\to\infty} 2 = 2$

A. ∞

$\boxed{\text{B. } 2}$

C. 0

D. Does not exist

(d) $\lim_{x\to 0+} x^2 \ln(x)$

$0 \cdot (-\infty)$ $\lim_{x\to 0^+} \frac{\ln x}{1/x^2}$ $\frac{-\infty}{\infty}$ L'H

$\lim_{x\to 0^+} \frac{1/x}{-2/x^3} = \lim_{x\to 0^+} -\frac{1}{2}x^2 = 0$

A. $-1/2$

B. ∞

C. $-3/2$

$\boxed{\text{D. } 0}$

(e) $\lim_{x\to\frac{\pi}{2}} \frac{\sin(x)}{\csc^2(x)}$

Plug in $\pi/2$ for X $\sin\frac{\pi}{2}=1$

$\csc\frac{\pi}{2}=1$

A. ∞

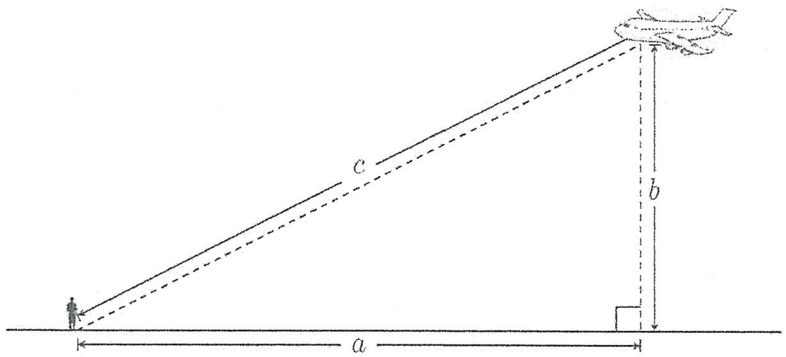
B. 0

$\boxed{\text{C. } 1}$

D. $\pi/2$

Answer is 1

13. (7 points) An airplane flies at a constant altitude of 3 miles and is heading toward a point directly over a stationary observer. Suppose that the speed of the plane is 400 miles per hour. Consider the figure below. Find the rate of change of the distance c, in miles per hour, of the plane from the observer when $c = 5$ miles. Show all your work and include units.



$c^2 = a^2 + b^2$ b is ALWAYS 3

$c^2 = a^2 + 9$

$2c\frac{dc}{dt} = 2a\frac{da}{dt}$ OR $c\frac{dc}{dt} = a\frac{da}{dt}$

$\frac{da}{dt} = -400$ mph when $c = 5$ miles

$5^2 = a^2 + 9$

$a^2 = 16$ $a = 4$ miles

$5 \cdot \frac{dc}{dt} = 4 \cdot (-400)$

$\frac{dc}{dt} = \frac{4(-400)}{5} = -320$ mph