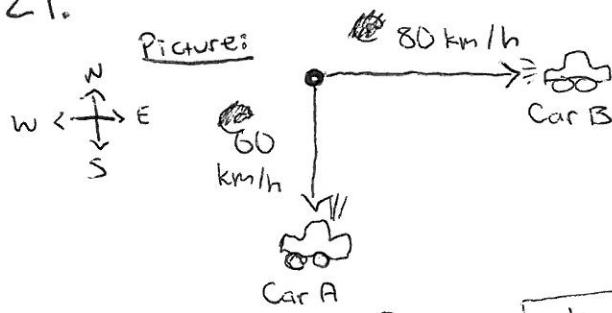


(Diagnostic Exam - Math 1300)

#21.

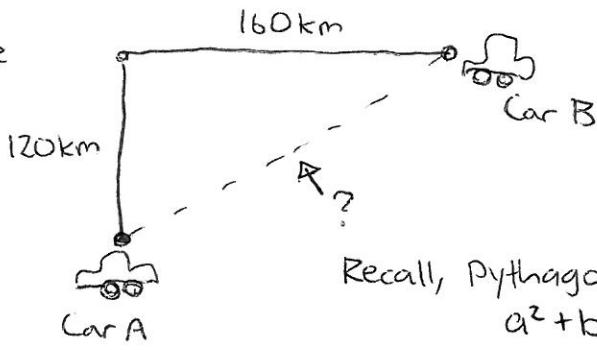


Recall $\boxed{\text{distance} = \text{rate} \cdot \text{time}}$.

After 2 hours, Car B has traveled $80 \frac{\text{km}}{\text{h}} \cdot 2 \text{ h} = 160 \text{ km}$ east.

After 2 hours, Car A has traveled $60 \frac{\text{km}}{\text{h}} \cdot 2 \text{ h} = 120 \text{ km}$ south.

So we have



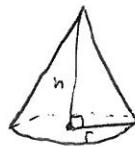
Recall, Pythagorean Theorem -

$$a^2 + b^2 = c^2$$

Then, Car A and Car B are $\sqrt{(120)^2 + (160)^2} = 200 \text{ km}$ apart.

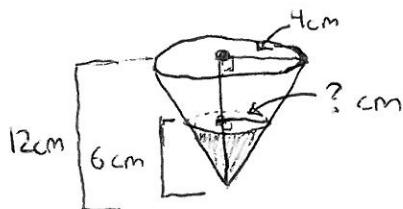
#22.

picture:



$$h = \text{height} = 12 \text{ cm}$$

$$r = \text{radius} = 4 \text{ cm}$$



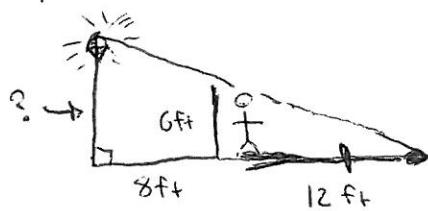
Notice that we have the triangles, and these are similar triangles.
So $\frac{12}{4} = \frac{6}{?}$, and we get $? = 2$.

So the radius of the ^{part of the} cone actually filled with water is 2 cm.

Using that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, we get the volume of water is $\frac{1}{3}\pi (2)^2 (6)$, or $8\pi \text{ cm}^3$.

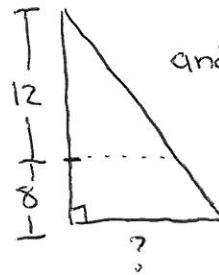
#23:

picture:



Using the idea of similar triangles,
we see we have two triangles

here



$$\text{So } \frac{12}{6} = \frac{(12+8)}{?}, \text{ and we get } ? = 10 \text{ ft.}$$

That is, the light pole is 10 ft. tall.

#24. To find the y-intercept of $x^3 - 4x^2 + 12x - 25$,

we plug in 0 for x. We get that

$$(0)^3 - 4(0)^2 + 12(0) - 25 = -25 \text{ is the y-value}$$

of the y-intercept. The point is then $(0, -25)$.

#25. For $\left| \frac{5-3x}{4} \right| < 5$, enter ~~$\frac{5-3x}{4} > 5$~~

$$\text{we need } -5 < \frac{5-3x}{4} \text{ and } \frac{5-3x}{4} < 5.$$

$$\text{Then } -20 < 5-3x \text{ and } 5-3x < 20,$$

$$\text{so } -25 < -3x \text{ and } -3x < 15.$$

When we divide by -3 to isolate x, we must flip the inequality, so we get

$$\frac{25}{3} > x \text{ and } x > -5. \text{ So } x \text{ is a solution}$$

$$\text{to } \left| \frac{5-3x}{4} \right| < 5 \text{ if } -5 < x < \frac{25}{3}, \text{ or } x \text{ is in}$$

the interval $(-5, \frac{25}{3})$.

#26. To find the distance between two points, recall the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \text{distance between } (x_1, y_1) \text{ and } (x_2, y_2)$.

Here, $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (0, 4)$.

$$\begin{aligned}\text{We get the distance} &= \sqrt{(-1-0)^2 + (2-4)^2} = \sqrt{(-1)^2 + (-2)^2} \\ &= \sqrt{1+4} = \sqrt{5}.\end{aligned}$$

#27. The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by $\frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$.

So the slope of the line passing through $(-5, -2)$ and $(1, 4)$ is $\frac{(-2) - (4)}{(-5) - (1)} = \frac{-6}{-6} = 1$.

#28. To factor $2x^3 + 8x^2 - 3x - 12$, notice that the ratio $2:8$ is the same as $3:12$.

This suggests we should look at $2x^3 + 8x^2$ and $-3x - 12$, and try to simplify these terms in such a way that they each become a product of two terms with one of these terms common to both:

$$2x^3 + 8x^2 = 2x^2(\cancel{1x} + 4) \text{ and } -3x - 12 = -3(\cancel{1x} + 4).$$

$$\text{So } 2x^3 + 8x^2 - 3x - 12 = 2x^2(1x + 4) - 3(1x + 4).$$

$$\text{This equals } (2x^2 - 3)(1x + 4).$$

#29. Using the rules of logarithms,

we get that $\log_3(x+6) - \log_3(x-2) = 2$

becomes " $\log_3\left[\frac{(x+6)}{(x-2)}\right] = 2$ ".

This statement means $3^2 = \frac{(x+6)}{(x-2)}$.

So $9 = \frac{(x+6)}{(x-2)}$, and we get $(x-2) \cdot 9 = \frac{(x+6)}{(x-2)} \cdot (x-2)$.

Then $(x-2) \cdot 9 = x+6$, so $9x - 18 = x+6$.

We get $8x = 24$, so $x = 3$.

#30. Given $P(t) = 500(1.3)^t$, plugging in $t=0$,

we get that our initial population

is $P(0) = 500(1.3)^{(0)} = 500$.

We would like to find the time - call it t_{double} - such that $P(t_{\text{double}}) = 2 \cdot 500$. By our formula $P(t)$, we also know that $P(t_{\text{double}}) = 500(1.3)^{t_{\text{double}}}$.

So $500(1.3)^{t_{\text{double}}} = 2 \cdot 500$.

Then $(1.3)^{t_{\text{double}}} = 2$, so $\ln((1.3)^{t_{\text{double}}}) = \ln(2)$.

Using the rules of logarithms,

$$\ln((1.3)^{t_{\text{double}}}) = t_{\text{double}} \cdot \ln(1.3).$$

$$\text{So } t_{\text{double}} \cdot \ln(1.3) = \ln(2).$$

$$\text{Then } t_{\text{double}} = \frac{\ln(2)}{\ln(1.3)}.$$

#31. For a polynomial to have zeros at $x=-2$,
 $x=1$, and $x=3$, we must be able to
plug ~~-2~~ -2, 1, and 3 into our polynomial
and get 0. We can get such a polynomial
by $(x+2)(x-1)(x-3) = (x^2+x-2)(x-3)$
 $= (x^3 - 2x^2 - 5x + 6)$.