

**Algebra Prelim Syllabus**  
**Department of Mathematics**  
**University of Colorado**

**Group Theory.** Basic definitions and examples, lattice of subgroups/normal subgroups, quotient groups, isomorphism theorems, the characterization of products, Lagrange's Theorem, Cauchy's Theorem, Cayley's Theorem, the structure of finitely generated abelian groups, group actions, the class equation, Sylow's Theorems, the Jordan-Hölder Theorem, simple groups, solvable groups, semidirect products, free groups, presentations of groups.

**Ring Theory.** Basic definitions and examples, lattice of subrings/ideals, prime ideals, quotient rings, chain conditions, rings of fractions, Chinese Remainder Theorem, Euclidean domains, PID's, UFD's, polynomial rings, irreducibility criteria for polynomials.

**Modules and Linear Algebra.** Basic definitions and examples, lattice of submodules, quotient modules, the matrix of a linear transformation, minimal polynomial of a transformation, Cayley-Hamilton Theorem over a commutative ring, trace and determinant, dual spaces, modules over a PID, rational canonical form, Jordan canonical form.

**Field Theory.** Basic definitions and examples, field extensions, simple extensions, algebraic extensions, transcendental extensions, separable extensions, cyclotomic extensions, solution of the Greek construction problems, splitting fields and normality, algebraic closure, the Galois correspondence, Galois groups of extensions/polynomials, solvable and radical extensions, the insolvability of the quintic, Fundamental Theorem of Algebra, finite fields, Frobenius endomorphism.

**Miscellaneous.** Applications of Zorn's Lemma.

**Primary Reference.** D. Dummit and R. Foote, Abstract Algebra: Chapters 1–14 and Appendix I, but skipping sections 9.6, 10.4, 10.5, 11.5.

**Other Good references:** T. Hungerford, Algebra, N. Jacobson, Basic Algebra I, S. Lang, Algebra.

**Note on the teaching of the course:** It is useful to introduce the basic notions of categories and morphisms, functors and natural transformations, as inspiration, but this material will not be on the prelim.