MATH 2400: Calculus 3, Spring 2014
Midterm 3

April 9, 2014

NAME:

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

Circle your section.

001 J. Migler .... (9am)

002 T. Davison .... (10am)

003 I. Mishev .... (11am)

004 I. Mishev .... (12pm)

005 M. Walter .... (1pm)

006 S. Andrews .... (2pm)

You must show all of your work. Please write legibly and box your answers. The use of calculators, books, notes, etc. is not permitted on this exam. Please provide exact answers when possible. For example, if the answer is π, write the symbol "π" and not the decimal 3.14159. . . .

Question	Points	Score
1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
Total:	100	

1. (15 points) Find a parametrization $\vec{r}(t)$ for the line $y = -\frac{5}{3}x + 5$ such that $\vec{r}(0)$ is on the x-axis and $\vec{r}(1)$ is on the y-axis.

2. (15 points) Let W be the three dimensional object that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and bounded above by the sphere $x^2 + y^2 + z^2 = 4$. Find the volume of W.

3. (15 points) Use the change of variables $s = x + y$, $t = y$ to compute the area of the elliptical region bounded by $x^2 + 2xy + 2y^2 = 1$.

4. Consider the vector field
$$\vec{F}(x,y) = y\vec{i} + x\vec{j}.$$

   (a) (10 points) Show that for any constant c, the curve parameterized by
$$\vec{r}(t) = c(e^t + e^{-t})\vec{i} + c(e^t - e^{-t})\vec{j}$$
   is a flow line for $\vec{F}$.

   (b) (10 points) Find a parametrization for the flow line for $\vec{F}$ that passes through the point $(1,0)$, i.e. solve for c.

5. Consider the vector field
$$\vec{F}(x,y) = \left(\frac{y}{1+x^2y^2}\right)\vec{i} + \left(\frac{x}{1+x^2y^2} + 1\right)\vec{j}.$$

   (a) (10 points) Let C be the segment of the hyperbola $y = \frac{1}{x}$ from $(1,1)$ to $(2, \frac{1}{2})$. Evaluate the line integral
$$\int_C \vec{F} \cdot d\vec{r}$$
   by parameterizing C. Do **not** use the Fundamental Theorem of Calculus for Line Integrals.

(b) (5 points) Show that the function $f(x, y) = \arctan(xy) + y$ is a potential function for $\vec{F}$.

(c) (5 points) Use the Fundamental Theorem of Calculus for Line Integrals to evaluate

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the segment of the hyperbola $y = \frac{1}{x}$ from $(1, 1)$ to $(2, \frac{1}{2})$.

6. (15 points) Let C be the circle of radius 3 centered at the origin, oriented counterclockwise. Define

$$\vec{F}(x, y) = -x^2 y\vec{i} + y^2 x\vec{j}.$$

Calculate the circulation of $\vec{F}$ around C by using Green's Theorem.