MATH 2400: Calculus 3, Spring 2014
Final Exam

May 6, 2014

NAME:

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

Circle your section.

001 J. Migler .... (9am)

002 T. Davison .... (10am)

003 I. Mishev .... (11am)

004 I. Mishev .... (12pm)

005 M. Walter .... (1pm)

006 S. Andrews .... (2pm)

You must show all of your work. Please write legibly and box your answers. The use of calculators, books, notes, etc. is not permitted on this exam. Please provide exact answers when possible. For example, if the answer is π, write the symbol "π" and not the decimal 3.14159. . . .

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	10	
7	15	
Total:	100	

1. Calculate the following integrals.

   (a) (5 points) $\int_0^2 \int_y^2 e^{-x^2} \, dx dy$.

(b) (10 points) $\iint_R (4x+3y)dA$, where R is the region bounded by the lines $y = x$, $y = x-2$, $y = -2x$, and $y = 3-2x$ (hint: use the transformations $x = \frac{1}{3}(u+v)$ and $y = \frac{1}{3}(v-2u)$).

2. (15 points) Let C be the curve in the yz plane defined by $z = 5\sqrt{y-1}$ for $1 \leq y \leq 2$. Parameterize the surface obtained by rotating C about the z-axis; be sure to include the bounds for your parameters.

3. Let

$$f(x, y) = \frac{x^3}{3} - \frac{y^2}{2} - xy - 1.$$

(a) (6 points) Find all the critical points of f.

(b) (9 points) Classify each critical point as a local maximum, local minimum, or saddle point.

4. (15 points) Use Stokes' Theorem to calculate the circulation of the vector field

$$\vec{F}(x, y, z) = 6xz\vec{i} + (4x + 7yz)\vec{j} + 6x^2\vec{k}$$

around the circle $x^2 + y^2 = 9$, $z = 0$, oriented counterclockwise when viewed from above (i.e. from the positive z-axis).

5. Given the vector field $\vec{F}(\vec{r}) = \dfrac{\vec{r}}{\| \vec{r} \|^3}$ (for $\vec{r} \neq \vec{0}$) do the following.

   (a) (5 points) Show that $\nabla \cdot \vec{F}(\vec{r}) = 0$ for $\vec{r} \neq \vec{0}$. (Note this is the divergence of the vector field $\vec{F}$.)

(b) (5 points) Compute $\int_{S_1} \vec{F} \cdot d\vec{A}$, where S_1 is a sphere of radius 1, centered at $(0,0,0)$, and oriented outwards. (Note that the Divergence Theorem does not apply!)

(c) (5 points) Compute $\int_{S_2} \vec{F} \cdot d\vec{A}$, where S_2 is the ellipsoid given by $x^2 + y^2 + 4z^2 = 16$, oriented outwards. (Hint: use the Divergence Theorem along with your answer to part (b).)

6. Let $f(x, y, z) = \sin(zy) + 3x^2 z$.
   (a) (5 points) Calculate $\operatorname{grad} f$.

   (b) (5 points) Calculate $\operatorname{curl}(\operatorname{grad} f)$.

7. (15 points) Let $\vec{F}(x, y, z) = \langle x^2, y, \sin(z)\rangle$. Calculate

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the portion of the helix $\vec{r}(t) = \langle \cos(t), \sin(t), t\rangle$ from $t = 0$ to $t = \pi$.