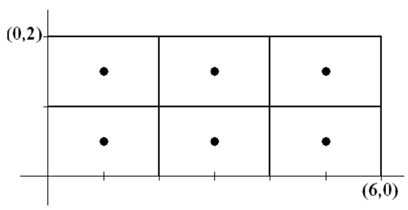
Midterm 3 Review

Short Answer

2. Give an example of a non-constant function $f(x, y)$ such that the average value of f over $R = \{(x,y) \mid -1 \le x \le 1, -1 \le y \le 1\}$ is 0.

3. Compute the Riemann sum for the double integral $\iint_R x + 2y \, dA$ where $R = [0, 6] \times [0, 2]$ for the given grid and choice of sample points.



4. Evaluate $\iint_R 3 \, dA$ where $R = [-1, 1] \times [2, 3]$ by first identifying it as the volume of a solid.

5. Evaluate $\iint_R \sqrt{4 - x^2} \, dA$ where $R = [-2, 2] \times [0, 3]$ by first identifying it as the volume of a solid.

6. Find $\int_0^2 f(x,y) \, dy$ and $\int_0^1 f(x,y) \, dx$ for $f(x,y) = 2xy - 3x^2$.

7. Calculate the double integral $\iint_R xy^2 + \frac{y}{x} \, dA$, where $R = \{(x,y) \mid 2 \le x \le 3, -1 \le y \le 0\}$.

8. Compute the average value of $f(x,y) = 3 + xy^2$ over $R = [-2, 2] \times [0, 1]$.

9. Compute the average value of $f(x,y) = x^2 + y^2$ over $R = \{(x,y) \mid 0 \le x \le 2, 0 \le y \le 2\}$.

10. Evaluate the double integral of ye^{y^4} over the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$.

11. Evaluate $\int_0^1 \int_{2x}^2 \frac{1}{\sqrt{1 + (2x/y)}} \, dy \, dx$.

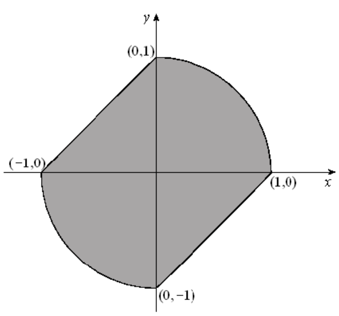
12. Evaluate $\int_0^1 \int_{x+1}^2 e^{x/(y-1)}\, dy\, dx$.

13. Find the volume of the solid that is common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

14. Express the integral $\int_0^2 \int_{x^2}^4 (xy^2 + x)\, dy\, dx$ as an equivalent integral with the order of integration reversed.

15. Let E be the solid under the plane $x + y + z = 5$ and above the region in the xy-plane bounded by $x = 4 - y^2$ and $x + y = 2$. Express the volume of E as an iterated integral in rectangular coordinates.

16. Rewrite $\iint_R f(x,y)\, dA$ as an iterated integral with y as the variable of integration in the outer integral, where R is the region shown below.



17. Use polar coordinates to evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dy\, dx$.

18. Convert the integral $\int_0^2 \int_{-\sqrt{4-x^2}}^0 \frac{xy}{\sqrt{x^2+y^2}}\, dy\, dx$ to polar coordinates and evaluate it.

19. Rewrite the integral $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} x\, dx\, dy$ in terms of polar coordinates, then evaluate the integral.

20. Rewrite the integral $\int_0^1 \int_x^{1+\sqrt{1-x^2}} x\, dy\, dx$ in terms of polar coordinates, then evaluate the integral.

21. Let R be the region bounded by $y = x^2$, $y = 0$, and $x = 1$. Find the center of mass of a lamina in the shape of R with density function $p(x,y) = xy$.

22. Find the moment of inertia I_x about the *x*-axis and the moment of inertia I_y about the *y*-axis for the region in the first quadrant bounded by *y* = *x* and $y^2 = x^3$, assuming *p* = 1.

23. Find the mass and center of mass of the lamina that occupies the triangular region with vertices (0, 0), (1, 1), and (4, 0), and has density function $p(x, y) = x$.

24. Find the center of mass of the lamina that occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant if the density at any point is proportional to the square of its distance from the origin.

26. Find the area of the part of the surface $z = x + y^2$ that lies above the triangle with vertices (0, 0), (1, 1), and (0, 1).

27. Compute the area of that part of the graph of $3z = 5 + 2x^{3/2} + 4y^{3/2}$ which lies above the rectangular region in the first quadrant of the *xy*-plane bounded by the lines *x* = 0, *x* = 3, *y* = 0, and *y* = 6.

28. Find the area of the surface cut from the cone $z = 1 - \sqrt{x^2 + y^2}$ by the cylinder $x^2 + y^2 = y$.

29. Find the area of that part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane *z* = 1.

30. Find the area of that part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

31. Find the area of the surface with vector equation **r** $(s, t) = \langle s\cos t, s\sin t, s \rangle$, $1 \leq s \leq 5$, $0 \leq t \leq 2\pi$.

32. Evaluate the iterated integral $\int_0^2 \int_0^{x^2} \int_0^{\ln z} xe^y \, dy\, dz\, dx$.

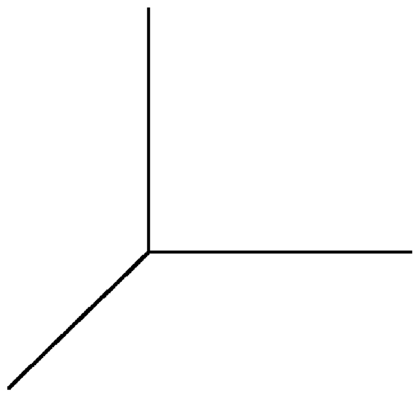
33. Find $\iiint_S x^2 y \, dV$, where *S* is the solid bounded by the cylinder $y = x^2$ and the planes *z* = 0, *y* = 1, and *z* = *y*.

34. Evaluate the iterated integral $\int_0^{\pi/2} \int_y^{\pi/2} \int_0^{xy} \cos\left(\frac{z}{y}\right) dz\, dx\, dy$.

35. Find the volume of the solid formed by the intersection of the cylinder $y = x^2$ and the two planes given by *z* = 0 and *y* + *z* = 4.

36. Suppose the volume of a solid is given by $V = \int_0^3 \int_0^{(3-z)/2} \int_0^{4-x^2} dy\,dx\,dz$.

(a) Sketch the solid whose volume is given by V.



(b) Evaluate the integral to find the volume of the solid.

37. Sketch the solid whose volume is given by the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{2-x^2-y^2}} dz\,dy\,dx$.

38. Find the average value of the function $f(x, y, z) = xyz$ over the solid E bounded by planes $z = y$, $y = x$, $x = 1$, and $z = 0$.

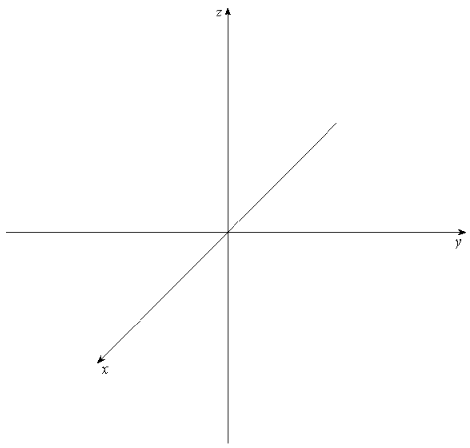
39. Find the z-coordinate of the centroid of the solid E bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$.

40. Find the volume bounded above by the surface $z = x^2 - y^2$, $x \geq 0$, below by the xy-plane, and laterally by the cylinder $x^2 + y^2 = 1$.

41. Find the volume of the region inside the cylinder $x^2 + y^2 = 7$ which is bounded below by the xy-plane and above by the sphere $x^2 + y^2 + z^2 = 16$.

42. Evaluate $\iiint_E e^{(x^2+y^2+z^2)^{3/2}}\,dV$, where E is the solid bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \frac{1}{\sqrt{3}}\sqrt{x^2 + y^2}$.

43. Sketch the region E whose volume is given by the integral $\int_0^{2\pi} \int_{\pi/6}^{5\pi/6} \int_{1/\sin\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.



44. Find the mass of a solid ball of radius 2 if the density at each point (x, y, z) is $\dfrac{3}{1+\sqrt{x^2+y^2+z^2}}$.

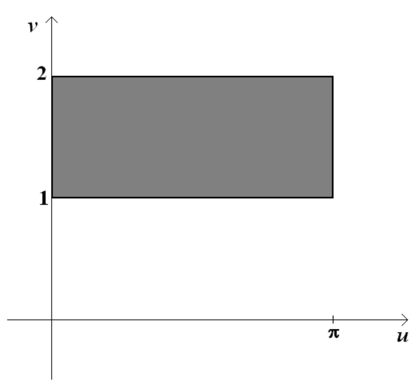
45. Use the change of variables $x = au$, $y = bv$, $z = cw$ to evaluate $\iiint_E y\,dV$, where E is the solid enclosed by the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$.

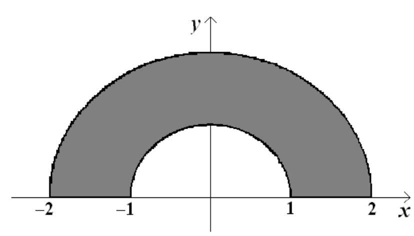
46. Compute the Jacobian of the transformation T given by $x = \sqrt{2}u - \dfrac{1}{\sqrt{2}}v$, $y = \dfrac{1}{\sqrt{2}}u + \sqrt{2}v$. Compute the area of the image of $S = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$ and compare it to the area of S.

47. Compute the Jacobian of the transformation T given by $x = v\cos 2\pi u$, $y = v\sin 2\pi u$. Describe the image of $S = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$, and compute its area.

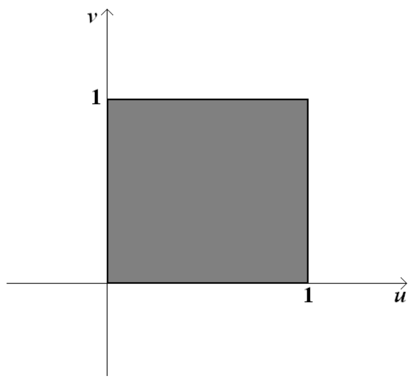
48. Evaluate $\iint_R \sqrt{b^2x^2+a^2y^2}\;dA$, where R is the region enclosed by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

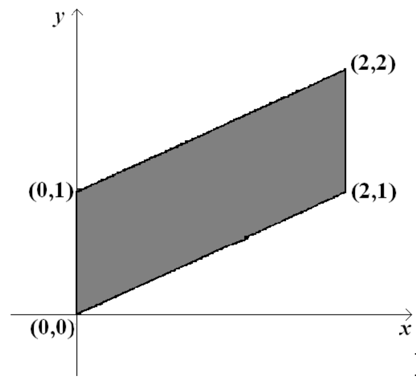
49. Find a transformation $x = x(u, v)$, $y = y(u, v)$ maps the region in the uv-plane into the xy-plane.





50. Find a transformation $x = x\,(u, v)$, $y = y\,(u, v)$ maps the region in the uv-plane into the xy-plane.





Midterm 3 Review

Answer Section

SHORT ANSWER

1. ANS:

$$\frac{87}{2}$$

PTS: 1

2. ANS:

Possible functions include $f\,(x, y) = x$ and $f\,(x, y) = y$. Any linear function of x and y with no constant term will work, as will many other functions.

PTS: 1

3. ANS:

60

PTS: 1

4. ANS:

6

PTS: 1

5. ANS:

6π

PTS: 1

6. ANS:

$$\int_0^2 f(x,y)\,dy = 4x - 6x^2, \int_0^1 f(x,y)\,dx = y - 1$$

PTS: 1

7. ANS:

$$\frac{5}{6} + \ln\sqrt{\frac{2}{3}}$$

PTS: 1
8. ANS:
3

PTS: 1
9. ANS:
$\frac{8}{3}$

PTS: 1
10. ANS:
$\frac{e^{16}-1}{4}$

PTS: 1
11. ANS:
$2\sqrt{2}-2$

PTS: 1
12. ANS:
$\frac{e-1}{2}$

PTS: 1
13. ANS:
$\frac{16a^3}{3}$

PTS: 1
14. ANS:
$\int_0^4 \int_0^{\sqrt{y}} (xy^2 + x)\,dx\,dy$

PTS: 1
15. ANS:
$\int_{-1}^2 \int_{2-y}^{4-y^2} (5-x-y)\,dx\,dy$

PTS: 1
16. ANS:
$\int_{-1}^0 \int_{-\sqrt{1-y^2}}^{y+1} f(x,y)\,dx\,dy + \int_0^1 \int_{y-1}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

PTS: 1

17. ANS:
$\frac{e-1}{4e}\pi$

PTS: 1
18. ANS:
$-\frac{4}{3}$

PTS: 1
19. ANS:
$\int_{-\pi/2}^{\pi/2}\int_0^1 r\cos\theta\, r\, dr\, d\theta = \frac{2}{3}$

PTS: 1
20. ANS:
$\int_{\pi/4}^{\pi/2}\int_0^{2\sin\theta} r\cos\theta\, r\, dr\, d\theta = \frac{1}{2}$

PTS: 1
21. ANS:
$\left(\frac{6}{7}, \frac{1}{2}\right)$

PTS: 1
22. ANS:
$I_x = \frac{1}{44}$; $I_y = \frac{1}{36}$

PTS: 1
23. ANS:
$m = \frac{10}{3}; (\bar{x}, \bar{y}) = (2.1, 0.3)$

PTS: 1
24. ANS:
$(x, y) = \left(\frac{8}{5\pi}, \frac{8}{5\pi}\right)$

PTS: 1
25. ANS:
$C = 2$

PTS: 1
26. ANS:

$\frac{3}{\sqrt{6}} - \frac{1}{3\sqrt{2}}$

PTS: 1
27. ANS:
$\frac{4}{15}(392\sqrt{7} - 789)$

PTS: 1
28. ANS:
$\frac{\pi\sqrt{2}}{4}$

PTS: 1
29. ANS:
4π

PTS: 1
30. ANS:
4π

PTS: 1
31. ANS:
$24\sqrt{2}\,\pi$

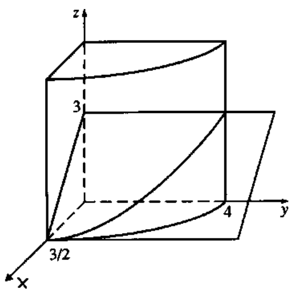
PTS: 1
32. ANS:
$\frac{4}{3}$

PTS: 1
33. ANS:
$\frac{4}{27}$

PTS: 1
34. ANS:
$\frac{\pi}{2} - 1$

PTS: 1
35. ANS:
$\frac{256}{15}$

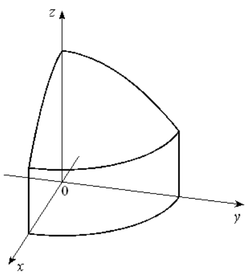
PTS: 1
36. ANS:



(a)

(b) $\frac{261}{32}$

PTS: 1
37. ANS:



PTS: 1
38. ANS:
$\frac{1}{8}$

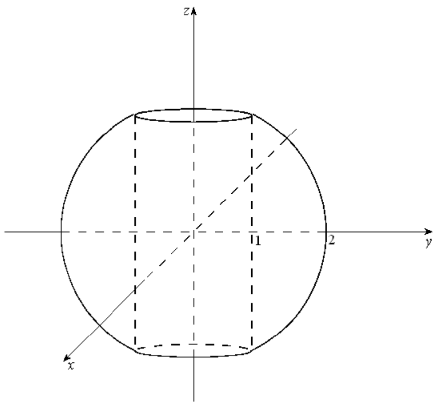
PTS: 1
39. ANS:
$z = \frac{3}{2}$

PTS: 1
40. ANS:
$\frac{1}{2}$

PTS: 1
41. ANS:
$\frac{74\pi}{3}$

PTS: 1
42. ANS:
$\frac{\pi(e-1)}{3}$

PTS: 1
43. ANS:
A sphere of radius 2 with a hole of radius 1 drilled through the center.



PTS: 1
44. ANS:
$12\pi \ln 3$

PTS: 1
45. ANS:
0

PTS: 1
46. ANS:
$J = \frac{5}{2}$, area of image = $\frac{5}{2}$

PTS: 1
47. ANS:
$J = \pi$, image of $S = \{(x,y) \mid x^2 + y^2 \leq 1\}$ area of image = π

PTS: 1
48. ANS:
$$\frac{2\pi(ab)^2}{3}$$

PTS: 1
49. ANS:
$x = v \cos u$, $y = v \sin u$

PTS: 1
50. ANS:
$x = 2u$, $y = u + v$

PTS: 1