Math 2400: Midterm 3 Review

Disclaimer: By no means is this review to be considered complete. You are responsible for all material from class, written and online homework, and the book.

1. Compute $\int\int_D \sqrt{1-x^2-y^2}dA$, where D is the disk $x^2+y^2 \leq 1$.
2. Evaluate the integral
$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2 dxdy.$$
3. Find the volume of the torus defined by the equation $\rho = \sin(\phi)$.
4. Calculate $\int\int_R (3x+4y)dA$, where R is the region bounded by the lines $y = x$, $y = x-2$,$y = -2x$ and $y = 3-2x$ (hint: use the transformations $x = \frac{1}{3}(u+v)$ and $y = \frac{1}{3}(v-2u)$).
5. Let $\vec{F}(x,y) = \langle -x^2, 2xy\rangle$. Find an equation for the curve that goes through $(1,2)$ and is perpendicular to $\vec{F}$ at every point.
6. The motion of a particle is given by the parametric equation
$$\vec{r}(t) = \langle t^3-3t, t^2-2t, t+5\rangle.$$
Give an equation for the tangent line to the path of the particle at time $t = -2$.
7. Adapt the parametrization of the sphere to find a parametrization of the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$
8. Find a parametric equation of the line passing through the points $(1,2,3)$ and $(3,5,7)$ and find the shortest distance from the line to the origin.
9. Calculate the Jacobian for each of the following changes of variables.
   (a) Cartesian to cylindrical.
   (b) Cartesian to spherical.
   (c) Cylindrical to spherical.
10. Where does the curve parametrized by
$$\vec{r}(t) = \langle t^2, 2t+1, 1-t^2\rangle$$
intersect the plane $-9x-2y-10z = 0$?
11. Let $\vec{F} = -y\vec{i} + x\vec{j}$.
   (a) Calculate $\int_C \vec{F}\cdot d\vec{r}$, where C is the upper half of the circle $x^2+y^2 = 4$, oriented clockwise.
   (b) Calculate $\int_C \vec{F}\cdot d\vec{r}$, where C is the line segment from $(-2,0)$ to $(2,0)$.
   (c) Based on your answers to (a) and (b), what can you conclude about $\vec{F}$?

12. Let $\vec{F} = x\vec{i} + y^3\vec{j} - \sin(z)\vec{k}$. Find
$$\int_C \vec{F} \cdot d\vec{r},$$
where C is the portion of the helix $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ from $(1, 0, 0)$ to $(-1, 0, \pi)$.

13. Find an equation of the line of intersection of the planes $x + y + z = 3$ and $x - 2y + 3z = 0$.

14. Find a parametric equation of the plane containing the points $(1, 2, 3)$, $(-3, 5, -7)$ and $(8, 12, 0)$.

15. Find a potential function f for the vector field
$$\vec{F} = (x^2 + y)\vec{i} + (x + \cos(z))\vec{j} + (z - y\sin(z))\vec{k}.$$