

MIDTERM 1 REVIEW

Disclaimer: By no means is this review to be considered complete. You are responsible for all materials from class, written and online homework, and the book.

- (1) Suppose $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors in $\mathbb{R}^3$ such that $\vec{a} + \vec{b} + \vec{c} = 0$. Show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$.
- (2) Describe geometrically with words and sketch the solutions to the following equations
 - (a) $x = 5$ in $\mathbb{R}^1, \mathbb{R}^2$ and $\mathbb{R}^3$
 - (b) $(x - 1)^2 + y^2 = 36$ in $\mathbb{R}^2$ and $\mathbb{R}^3$
 - (c) $z = y^2$ in $\mathbb{R}^3$
 - (d) $y = 8z - 3x + 1$ in $\mathbb{R}^3$
- (3) Using the concept of vectors, find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$ in $\mathbb{R}^2$. The distance from a point P to a line L is defined to be length of the shortest line which connects P to the line L . Hint: Draw a picture of the shortest line. What do you notice about it?
- (4) Sketch contour diagrams of the following functions.
 - (a) $z = f(x, y) = 8x - 2y$
 - (b) $z = f(x, y) = \frac{x-y}{y}$
- (5) Suppose $z = f(x, y)$. Can level curves for different z -values overlap (i.e. contain the same point)? Does (4b) contradict this?
- (6) Determine which of the following functions are continuous at the given points. Be sure to justify your answer.
 - (a) The function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto f(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^6} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

at the point $(0, 0)$.

- (b) The function

$$g : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto g(x, y) = (x^2 + y^2)^{3/2}$$

at the point $(0, 0)$.

- (c) The function

$$h : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto h(x, y) = x + 5y$$

at the point $(1, 2)$.

- (7) Let S be the sphere of radius 4 centered at $(1, 1, \sqrt{3})$.
 - (a) Write down an equation for S .
 - (b) Verify that $P = (1 + \sqrt{3}, 2, 3\sqrt{3})$ lies on S .
 - (c) Determine the plane tangent to the sphere at the point P . Hint: How do vectors from the center of the sphere to a point on the sphere relate to the tangent plane at that point?

- (8) Let $\vec{u} = \langle 1, 0, 1, -5 \rangle$ and $\vec{v} = \langle -2, 101, 7, 1 \rangle$ be vectors in $\mathbb{R}^4$. Are they parallel? Are they perpendicular? Can you use the cross product to produce a vector perpendicular to both of them?
- (9) Find the angle between a diagonal of a cube and one of its edges.
- (10) Consider the plane $z = 3x - y + 11$.
- Find a unit normal vector to the plane pointing downward. Call this vector $\vec{u}$.
 - Verify that $P = (0, 0, 11)$, $Q = (0, 1, 10)$ and $R = (1, 0, 14)$ are points on the plane.
 - Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$. Call this vector $\vec{v}$.
 - Compute the cross product $\vec{u} \times \vec{v}$. Explain your answer geometrically.
- (11) Find the area of the (simple) quadrilateral with vertices $(1, 1)$, $(6, 3)$, $(7, 6)$ and $(2, 4)$ using concepts from class.