MATH 2400: Calculus III, Fall 2013
MIDTERM #2

October 16, 2013

YOUR NAME:

Circle Your Correct Section

001 E. Angel .... (9am)
002 E. Angel .... (10am)
003 A. Nita .... (11am)
004 K. Selker .... (12pm)
005 I. Mishev .... (1pm)
006 C. Farsi .... (2pm)
007 R. Rosenbaum .... (3pm)
008 S. Henry .... (9am)

Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed. No books, notes, etc. Throughout this exam, please provide exact answers where possible. That is: if the answer is 1/2, do not write 0.499 or something of that sort; if the answer is π, do not write 3.14159.

Problem	Points	Score
1	16	
2	17	
3	16	
4	17	
5	17	
6	17	
TOTAL	100	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

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1. Consider the function $f(x, y) = |xy|^{\frac{1}{2}} + x$.

(a) **(6 points)** By using the definition of directional derivative, show that $f_x(0, 0)$ and $f_y(0, 0)$ exist.

(b) **(5 points)** By using the definition of directional derivative, determine if the directional derivative $f_{\vec{u}}(0, 0)$ of f at $(0, 0)$ in the direction of $\vec{u} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ exists.

(c) **(5 points)** Compute $(f_x(0, 0)\vec{i} + f_y(0, 0)\vec{j}) \cdot \vec{u}$.

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2. You want to design a closed rectangular box at a minimum of cost. The box must have a volume of 60 cubic inches. The price per square inch for the box material is 3 cents for the top and bottom, 2 cents for the front and back, and 1 cent for the sides.

(a) **(2 points)** Does the extreme value theorem guarantee the existence of a box that minimizes the cost? Explain why or why not.

(b) **(15 points)** Find the dimensions of the box that minimize the cost.

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3. Let $h(t) = f(x(t), y(t))$, and denote by $\nabla f(x, y)$ the gradient of f at the point (x, y).

(a) **(8 points)** Use the multivariable chain rule to show that the derivative $h'(t)$ with respect to t of the function $h(t)$ is given by the following dot product

$$h'(t) = \nabla f(x(t), y(t)) \cdot (x'(t)\vec{i} + y'(t)\vec{j}\,).$$

(b) **(8 points)** Suppose now that for all t, $f(x(t), y(t)) = c$ where c is some constant. Use part (a) to show that $\nabla f(x(t), y(t))$ and $x'(t)\vec{i} + y'(t)\vec{j}$ are perpendicular for all t. Hint: Differentiate both sides with respect to t.

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4. (17 points) For the function $f(x, y) = x^3 + y^2 - 3x^2 + 10y - 7$, find and classify its critical points as local maxima, local minima, saddle points, or none of the above.

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5. You are standing on a mountain whose shape is described by the graph of the function $z = -2x^2 - 3y^2 + 1500$. You are at the point $(5, 10, 1150)$.

(a) **(9 points)** If you decide to walk up the mountain along the steepest path, what is the slope of that path at the point $(5, 10, 1150)$?

(b) **(8 points)** You instead decide to walk around the mountain in a clockwise direction along the contour $z = 1150$. In which direction (as a unit vector in the xy plane) should you start moving?

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6. (a) **(7 points)** SET UP BUT DO NOT EVALUATE an integral that gives the volume of the solid that is bounded below by the sphere $x^2 + y^2 + (z-4)^2 = 16$ and above by the cone $z = 8 - \sqrt{3(x^2 + y^2)}$. We require a very detailed answer. In particular, if you give your answer as a double or triple integral, give very explicit formulas describing the integral's domain, together with the precise function(s) to be integrated. If your answer is in terms of nested one–variable integrals, clearly label both end–functions of each integral, together with the precise function(s) to be integrated.

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(b) Consider the integral $\int_0^1 \int_0^{\frac{\sqrt{1-x}}{\sqrt{3}}} e^{-y^3+y} dy dx$

(i) **(3 points)** Draw and label the boundaries of the region of integration of the above integral.

(ii) **(7 points)** Evaluate the above integral.