1. (10) Let

$$f(x, y) = x^2 + 2y^2.$$

(i) Compute the directional derivative of f at $(1, \frac{1}{2})$ in the direction of the vector $\langle 3, 4 \rangle$.

(ii) Find the unit vector $\mathbf{u}$ where $D_{\mathbf{u}} f(1, \frac{1}{2})$ attains its maximum value.

2. (20) Find the minimum distance on the surface $xyz = 8$ from the point $(0, 0, 0)$.
Hint: You may use the method of Lagrange multipliers; it is easier if you minimize the *square* of the distance to $(0, 0, 0)$.

3. (20) Let R be the region in the xy-plane bounded by the curves $y = x^2 + 1$ and $y = 9 - x^2$.

(i) Set up (but do *not* evaluate) an integral to compute the volume of the solid region bounded by the surfaces $z = x^2 + y^2$ and $z = 10$ over the region R.

(ii) Use double integration to find the area A of R.

Editorial note: Do we really mean "use double integration", or "using double integration, or otherwise"? What if they use Calc 1 techniques instead?

4. (15) Set up (but do *not* evaluate) integrals to compute the following volumes:

(i) The solid bounded by the surfaces $z = x$, $x = y^2$, $x = 4$, and $z = 0$;

(ii) The region bounded by the paraboloid $y = x^2 + 2z^2$ and the parabolic cylinder $y = 2 - x^2$;

(iii) The solid bounded by the surfaces $x = z^2$, $x = 8 - z^2$, $x + y = 1$ and $y = 0$.

5. (15) **Editorial note: obviously this question is far too long, and we need to discuss what to cut out. Also, what does "graph" mean in part (iv)?**

(i) Find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$ in both cylindrical and spherical coordinates.

(ii) Find the volume of the solid bounded above by the sphere, $x^2 + y^2 + z^2 = 2$ and bounded below by the paraboloid $z = x^2 + y^2$.

(iii) Find the volume of the region that lies inside both the sphere, $x^2 + y^2 + z^2 = 4$ and the cylinder, $x^2 + y^2 - 2y = 0$.

(iv) Graph the surface $\rho = 2 + \cos\phi$ and compute the volume it holds.

6. (20)

(i) What is the graph of $r = 2\cos\theta$ on the xy-plane ?

(ii) Find a polar coordinate equation for the vertical line passing through the point $(1, 0)$ on the xy-plane.

(iii) Find the volume of the solid under the surface of $z = \dfrac{1}{\sqrt{x^2+y^2}}$ and above the region on the xy-plane bounded by the right-hand half of the circle $x^2 + y^2 = 2x$ and the vertical line passing through the point $(1, 0)$ on the xy-plane.
Hint: $\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta|$.

Name: _____

Section: _____

University of Colorado

Mathematics 2400: Third Midterm Exam

November 9, 2005

Problem	Points	Score
1	10	
2	20	
3	20	
4	15	
5	15	
6	20	
Total	100	