

**Math 4001 Analysis 2**  
**Homework Set 7**

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**Problem 1:** Determine the integral

$$\int_{-\infty}^{\infty} e^{-x^2} dx.$$

Hint: First compute  $\int_{\mathbb{R}^2} e^{-(x^2+y^2)} dA$  and use this to derive a formula for  $\int_{-\infty}^{\infty} e^{-x^2} dx$ . (6P)

**Problem 2:** Use a double integral to find the area of the region enclosed by the lemniscate, which, in polar coordinates, is given by  $r^2 = 4 \cos(2\theta)$ .

Hint: Draw a picture. (6P)

**Problem 3:**

(a) Determine the Jacobian matrix and the Jacobian determinant for the transformation

$$x = 3s + t + 2u, \quad y = s + 5t - u, \quad z = 2s - t + u. \quad (2P)$$

(b) Determine the Jacobian matrix and the Jacobian determinant  $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$  for the transformation from cylindrical to cartesian coordinates. (2P)

(c) Determine the Jacobian matrix and the Jacobian determinant  $\frac{\partial(r,\theta,\varphi)}{\partial(x,y,z)}$  for the transformation from cartesian coordinates to spherical coordinates.

Hint: Use Problem 3. on Homework 4. (4P)