Abelian congruences in locally finite Taylor varieties

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Locally finite Taylor varieties form an especially important and broad class within universal algebra. This is essentially the largest class of locally finite varieties for which anything "interesting" can be said about the structure of its members. Many characterizations are known for this class. One under-used characterization, proved originally by Hobby & McKenzie, is that a locally finite variety is Taylor iff it has a *weak difference term*. Here a 3-ary term $d(x, y, z)$ is a weak difference term for a variety $\mathcal{V}$ if it satisfies Maltsev's identities $d(x, x, y) \approx y \approx d(y, x, x)$ whenever x and y are elements of a block of an abelian congruence in a member of $\mathcal{V}$.

In this tutorial I will explain how the existence of a weak difference term shapes the structure of abelian congruences. Adapting a construction of Hagemann and Herrmann for congruence modular varieties, I will show how the blocks of an abelian congruence θ contained in a single block of its centralizer "fit together" in a single abelian group determined by θ and the relevant centralizer block. When θ is minimal and the algebra is finite, the abelian group is in fact a vector space over a finite field determined by θ. This leads to a notion of "similarity" between subdirectly irreducible algebras with abelian monolith, as well as a fruitful analysis of critical rectangular relations, generalizing work of Freese, Kearnes & Szendrei, and Zhuk.