Lattice-theoretic principles for synthetic higher category theory

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$(\infty, 1)$-categories generalize ordinary categories by introducing n-morphisms in each dimension n, with all of them being invertible for $n > 1$. This makes ∞-category theory much more involved than ordinary 1-category theory, due to the large amount of additional bookkeeping involved. In recent years, synthetic accounts have been developed to at least partially overcome this problem. This led to the synthetic study of ∞-categories in other foundation systems than set theory, such as homotopy type theory (HoTT). Based on Riehl–Shulman's simplicial extension of HoTT, we show how to construct the universe of ∞-groupoids in that setting, and prove that it satisfies desired properties such as being an ∞-category and directed univalence which says that its geometric arrows correspond to functors between ∞-groupoids. Our system relies on the addition of new axioms about the underlying directed interval. We will discuss these in detail and argue how they can be used in our synthetic theory to give relatively elementary combinatorial and order-theoretic proofs of theorems that in traditional set-theoretic foundations requires the development of hundreds of pages of intricate machinery.

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