Algebraic structures defined by the finite condensation on linear orders

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The finite condensation $\sim_F$ is an equivalence relation defined on a linear order L by $x \sim_F y$ if and only if the set of points lying between x and y is finite. We define an operation $\cdot_F$ on linear orders L and M by $L \cdot_F M = (LM)/{\sim_F}$; that is, $L \cdot_F M$ is the lexicographic product of L and M modulo the finite condensation. If $L/{\sim_F} \cong 1$ for an infinite linear order L, then L is order-isomorphic to one of $\mathbb{N}, \mathbb{N}^*$, or $\mathbb{Z}$. We show that under the operation $\cdot_F$, the set $R = \{1, \omega, \omega^*, \zeta\}$ (where ω^* is the order type of the negative integers and ζ is the order type of $\mathbb{Z}$) forms a left rectangular band. Further, each of the ordinal elements of R defines, via left or right multiplication modulo the finite condensation, a weakly order-preserving map on the class of ordinals. We describe these maps' action on the ordinals whose Cantor normal form is of finite degree.