Algebraic properties of groups in lattice framework

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In the last few years, we have been investigating groups in the framework of their lattices of weak congruences. They can be understood as lattices of all normal subgroups of all subgroups of the given group, ordered by inclusion. These lattices are algebraic, and they offer information about numerous algebraic properties of groups, formulated in lattice terms. Using these lattice properties, we have characterized many classes of groups. Usually, we were given necessary and sufficient conditions that should be satisfied by the weak congruence lattice of a group G in order that G belongs to a particular class.

In this talk, we systematically present those lattice properties, explaining their equivalent algebraic nature. We show that all basic features of classes and varieties of groups (formulated algebraically as operators S, H, and P) can be analyzed and investigated in lattice terms. E.g., among subgroups properties, we define a lattice relation of being a normal subgroup; dealing with operator H, we identify all homomorphic images of the group and its subgroups within the same weak congruence lattice; we also give the lattice description of the internal direct product of subgroups. All kinds of chains and series of (normal) subgroups or systems have equivalent descriptions in lattice terms, etc. We describe the center of a group, the commutator subgroup, and its usage, closure operators among subgroups, etc.

In addition to the systematic description of the lattice terms related to algebraic properties of groups, we give in this context several new weak congruence lattice characterizations of some group classes.

A question remains: which group properties are essentially lattice

properties?

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