The Funayama envelope as the T_D-hull of a frame

Guram Bezhanishvili, Ranjitha Raviprakash, Anna Laura Suarez, Joanne Walters-Wayland

May, 2025 (University of Colorado, Boulder, BLAST 2025)

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee\{a \wedge s \mid s \in S\}$.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee \{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee \{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.
3. Let **Frm** be the category of frames and frame homomorphisms.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee \{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.
3. Let **Frm** be the category of frames and frame homomorphisms.

Each topological space X has the frame $\Omega(X)$ of its open sets.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee \{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.
3. Let **Frm** be the category of frames and frame homomorphisms.

Each topological space X has the frame $\Omega(X)$ of its open sets. If $f : X \to Y$ is a continuous map between spaces, we have the frame homomorphism $\Omega(f) : \Omega(Y) \to \Omega(X)$ given by $\Omega(f)(V) = f^{-1}[V]$ for every $V \in \Omega(Y)$.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee \{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.
3. Let **Frm** be the category of frames and frame homomorphisms.

Each topological space X has the frame $\Omega(X)$ of its open sets. If $f : X \to Y$ is a continuous map between spaces, we have the frame homomorphism $\Omega(f) : \Omega(Y) \to \Omega(X)$ given by $\Omega(f)(V) = f^{-1}[V]$ for every $V \in \Omega(Y)$.

This defines a contravariant functor $\Omega : \mathbf{Top} \to \mathbf{Frm}$, where **Top** is the category of topological spaces and continuous maps.

Frames

Definition

1. A frame is a complete distributive lattice L satisfying the join-infinite distributive law (JID): $a \wedge \bigvee S = \bigvee\{a \wedge s \mid s \in S\}$.
2. A frame homomorphism is a map between two frames preserving finite meets and arbitrary joins.
3. Let **Frm** be the category of frames and frame homomorphisms.

Each topological space X has the frame $\Omega(X)$ of its open sets. If $f : X \to Y$ is a continuous map between spaces, we have the frame homomorphism $\Omega(f) : \Omega(Y) \to \Omega(X)$ given by $\Omega(f)(V) = f^{-1}[V]$ for every $V \in \Omega(Y)$.

This defines a contravariant functor Ω : **Top** $\to$ **Frm**, where **Top** is the category of topological spaces and continuous maps.

Ω has a *right adjoint* pt : **Frm** $\to$ **Top**, which maps a frame to its space of completely prime filters.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

A frame is spatial if it is isomorphic to the lattice of opens of a topological space.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

A frame is spatial if it is isomorphic to the lattice of opens of a topological space.

Sob the full subcategory of **Top** consisting of sober spaces.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

A frame is spatial if it is isomorphic to the lattice of opens of a topological space.

Sob the full subcategory of **Top** consisting of sober spaces.

SFrm the full subcategory of **Frm** consisting of spatial frames.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

A frame is spatial if it is isomorphic to the lattice of opens of a topological space.

Sob the full subcategory of **Top** consisting of sober spaces.
SFrm the full subcategory of **Frm** consisting of spatial frames.

Theorem (Dowker-Papert)

The contravariant adjunction (Ω, pt) *restricts to a dual equivalence between* **Sob** *and* **SFrm**.

Sober spaces and spatial frames

A space X is sober if every completely prime filter is of the form $N(x) := \{U \in \Omega(X) \mid x \in U\}$ for some unique $x \in X$.

A frame is spatial if it is isomorphic to the lattice of opens of a topological space.

Sob the full subcategory of **Top** consisting of sober spaces.
SFrm the full subcategory of **Frm** consisting of spatial frames.

Theorem (Dowker-Papert)

The contravariant adjunction (Ω, pt) restricts to a dual equivalence between **Sob** *and* **SFrm**.

This means that a sober space X can be reconstructed from $\Omega(X)$.

T_D-spaces and T_D-spatial frames

Definition

A topological space X is a T_D-space if each point x is locally closed ($\{x\}$ is an intersection of an open set and a closed set).

T_D-spaces and T_D-spatial frames

Definition

A topological space X is a T_D-space if each point x is locally closed ($\{x\}$ is an intersection of an open set and a closed set).

Let $\mathbf{Top}_D$ be the full subcategory of $\mathbf{Top}$ consisting of T_D-spaces.

T_D-spaces and T_D-spatial frames

Definition

A topological space X is a T_D-space if each point x is locally closed ($\{x\}$ is an intersection of an open set and a closed set).

Let $\mathbf{Top}_D$ be the full subcategory of **Top** consisting of T_D-spaces.

Definition

A frame L is T_D-spatial if $L \cong \Omega(X)$ for some T_D-space X.

T_D-spaces and T_D-spatial frames

Definition

A topological space X is a T_D-space if each point x is locally closed ($\{x\}$ is an intersection of an open set and a closed set).

Let $\mathbf{Top}_D$ be the full subcategory of **Top** consisting of T_D-spaces.

Definition

A frame L is T_D-spatial if $L \cong \Omega(X)$ for some T_D-space X.

In general, for a T_D-space X, the spectrum $\mathrm{pt}(\Omega(\mathrm{X}))$ may not be a T_D-space.

T_D-spaces and T_D-spatial frames

Definition

A topological space X is a T_D-space if each point x is locally closed ($\{x\}$ is an intersection of an open set and a closed set).

Let $\mathbf{Top}_D$ be the full subcategory of **Top** consisting of T_D-spaces.

Definition

A frame L is T_D-spatial if $L \cong \Omega(X)$ for some T_D-space X.

In general, for a T_D-space X, the spectrum $\mathrm{pt}(\Omega(\mathrm{X}))$ may not be a T_D-space. We thus need to work with a different spectrum, which is always T_D.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Remark

Every slicing filter is a completely prime filter, so $\mathrm{pt_D(L)} \subseteq \mathrm{pt(L)}$.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Remark

Every slicing filter is a completely prime filter, so $\mathrm{pt_D(L)} \subseteq \mathrm{pt(L)}$. In fact, $\mathrm{pt_D(L)}$ is precisely the collection of locally closed points of $\mathrm{pt(L)}$.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Remark

Every slicing filter is a completely prime filter, so $\mathrm{pt_D(L)} \subseteq \mathrm{pt(L)}$. In fact, $\mathrm{pt_D(L)}$ is precisely the collection of locally closed points of $\mathrm{pt(L)}$. Therefore, $\mathrm{pt_D(L)}$ is a T_D-subspace of $\mathrm{pt(L)}$.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F, b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Remark

Every slicing filter is a completely prime filter, so $\mathrm{pt_D(L)} \subseteq \mathrm{pt(L)}$. In fact, $\mathrm{pt_D(L)}$ is precisely the collection of locally closed points of $\mathrm{pt(L)}$. Therefore, $\mathrm{pt_D(L)}$ is a T_D-subspace of $\mathrm{pt(L)}$.

Frame homomorphisms $f : L \to M$ in general do not yield well-defined maps $\mathrm{pt_D(M)} \to \mathrm{pt_D(L)}$.

Slicing filters and D-morphisms

Definition

A filter F in frame L is called slicing if it is prime and there exist $a, b \in L$ such that $a \notin F$, $b \in F$, and a is covered by b.

For a frame L, let $\mathrm{pt_D(L)}$ be the space of slicing filters.

Remark

Every slicing filter is a completely prime filter, so $\mathrm{pt_D(L)} \subseteq \mathrm{pt(L)}$. In fact, $\mathrm{pt_D(L)}$ is precisely the collection of locally closed points of $\mathrm{pt(L)}$. Therefore, $\mathrm{pt_D(L)}$ is a T_D-subspace of $\mathrm{pt(L)}$.

Frame homomorphisms $f : L \to M$ in general do not yield well-defined maps $\mathrm{pt_D(M)} \to \mathrm{pt_D(L)}$.

Definition

A frame homomorphism $f : L \to M$ is a D-morphism provided $f^{-1}(F)$ is a slicing filter of L for each slicing filter F of M.

T_D-duality

$\mathbf{Top}_D$ the full subcategory of **Top** consisting of T_D-spaces.

T_D-duality

$\mathbf{Top}_D$ the full subcategory of **Top** consisting of T_D-spaces.

$\mathbf{Frm}_D$ the wide subcategory of **Frm** whose morphisms are D-morphisms.

T_D-duality

Top$_D$ the full subcategory of **Top** consisting of T_D-spaces.

Frm$_D$ the wide subcategory of **Frm** whose morphisms are D-morphisms.

If $f : X \to Y$ is a continuous map between T_D-spaces, then the frame homomorphism $\Omega(f)$ is a D-morphism.

T_D-duality

- **$\mathbf{Top}_D$** the full subcategory of **Top** consisting of T_D-spaces.
- **$\mathbf{Frm}_D$** the wide subcategory of **Frm** whose morphisms are D-morphisms.

If $f : X \to Y$ is a continuous map between T_D-spaces, then the frame homomorphism $\Omega(f)$ is a D-morphism.

Thus, $\Omega : \mathbf{Top} \to \mathbf{Frm}$ restricts to a functor from $\mathbf{Top}_D$ to $\mathbf{Frm}_D$.

T_D-duality

$\mathbf{Top}_D$ the full subcategory of **Top** consisting of T_D-spaces.

$\mathbf{Frm}_D$ the wide subcategory of **Frm** whose morphisms are D-morphisms.

If $f : X \to Y$ is a continuous map between T_D-spaces, then the frame homomorphism $\Omega(f)$ is a D-morphism.

Thus, $\Omega : \mathbf{Top} \to \mathbf{Frm}$ restricts to a functor from $\mathbf{Top}_D$ to $\mathbf{Frm}_D$. Its right adjoint is $\mathrm{pt_D} : \mathbf{Frm_D} \to \mathbf{Top_D}$.

T_D-duality

$\mathbf{Top}_D$ the full subcategory of **Top** consisting of T_D-spaces.

$\mathbf{Frm}_D$ the wide subcategory of **Frm** whose morphisms are D-morphisms.

If $f : X \to Y$ is a continuous map between T_D-spaces, then the frame homomorphism $\Omega(f)$ is a D-morphism.

Thus, $\Omega : \mathbf{Top} \to \mathbf{Frm}$ restricts to a functor from $\mathbf{Top}_D$ to $\mathbf{Frm}_D$. Its right adjoint is $\mathrm{pt_D} : \mathbf{Frm}_\mathrm{D} \to \mathbf{Top}_\mathrm{D}$.

Let **$\mathbf{TD\text{-}SFrm}_D$** be the full subcategory of $\mathbf{Frm}_D$ consisting of T_D-spatial frames.

T_D-duality

$\mathbf{Top}_D$ the full subcategory of **Top** consisting of T_D-spaces.

$\mathbf{Frm}_D$ the wide subcategory of **Frm** whose morphisms are D-morphisms.

If $f : X \to Y$ is a continuous map between T_D-spaces, then the frame homomorphism $\Omega(f)$ is a D-morphism.

Thus, $\Omega : \mathbf{Top} \to \mathbf{Frm}$ restricts to a functor from $\mathbf{Top}_D$ to $\mathbf{Frm}_D$. Its right adjoint is $\mathrm{pt_D} : \mathbf{Frm_D} \to \mathbf{Top_D}$.

Let **$\mathbf{TD\text{-}SFrm}_D$** be the full subcategory of $\mathbf{Frm}_D$ consisting of T_D-spatial frames.

Theorem (Banaschewski-Pultr)

The contravariant adjunction $(\Omega, \mathrm{pt_D})$ restricts to a dual equivalence between $\mathbf{Top}_D$ and $\mathbf{TD\text{-}SFrm}_D$.

Drawbacks

$$\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathrm{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathrm{pt_D}"] \\
\mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"']
\end{tikzcd}$$

Drawbacks

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathsf{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] & \mathbf{Frm}_D \arrow[l, bend left, "\mathsf{pt_D}"] \\
\mathbf{Top}_D \arrow[u, equal] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"']
\end{tikzcd}
$$

- The pt and $\mathsf{pt_D}$ functors are in general different, even for T_D-spatial frames.

Drawbacks

$$\begin{tikzcd} \mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathrm{pt}"] \\ \mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow] & \mathbf{SFrm} \arrow[u, hook, "full"'] \end{tikzcd} \qquad \begin{tikzcd} \mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathrm{pt_D}"] \\ \mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"'] \end{tikzcd}$$

- The pt and $\mathrm{pt_D}$ functors are in general different, even for T_D-spatial frames.
- The T_D-duality is not a restriction of the $\Omega \dashv \mathrm{pt}$ adjunction.

Drawbacks

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathrm{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathrm{pt_D}"] \\
\mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"']
\end{tikzcd}
$$

- The pt and $\mathrm{pt_D}$ functors are in general different, even for T_D-spatial frames.
- The T_D-duality is not a restriction of the $\Omega \dashv \mathrm{pt}$ adjunction.
- Not all frame morphisms between T_D-spatial frames are D-morphisms.

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, "\Omega"] & \mathbf{Frm} \\
\mathbf{Top_D} \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, dashed, red, "nonfull"']
\end{tikzcd}
$$

McKinsey-Tarski approach

An alternative approach is to work with powerset algebras and interior operators (Kuratowski, 1922).

McKinsey-Tarski approach

An alternative approach is to work with powerset algebras and interior operators (Kuratowski, 1922). This was generalized to Boolean algebras with interior operators by McKinsey and Tarski (1944).

McKinsey-Tarski approach

An alternative approach is to work with powerset algebras and interior operators (Kuratowski, 1922). This was generalized to Boolean algebras with interior operators by McKinsey and Tarski (1944).

Definition

1. A McKinsey-Tarski algebra or an MT-algebra is a complete Boolean algebra with an interior operator $\Box$ (that is,
$$\Box 1 = 1,\ \Box(a \wedge b) = \Box a \wedge \Box b,\ \Box a \leq a, \text{ and } \Box a \leq \Box\Box a).$$

McKinsey-Tarski approach

An alternative approach is to work with powerset algebras and interior operators (Kuratowski, 1922). This was generalized to Boolean algebras with interior operators by McKinsey and Tarski (1944).

Definition

1. A McKinsey-Tarski algebra or an MT-algebra is a complete Boolean algebra with an interior operator $\Box$ (that is,
$$\Box 1 = 1,\ \Box(a \wedge b) = \Box a \wedge \Box b,\ \Box a \leq a, \text{ and } \Box a \leq \Box\Box a).$$
2. An MT-morphism between MT-algebras M and N is a complete Boolean homomorphism $h : M \to N$ satisfying $h(\Box_M a) \leq \Box_N h(a)$ for each $a \in M$.

McKinsey-Tarski approach

An alternative approach is to work with powerset algebras and interior operators (Kuratowski, 1922). This was generalized to Boolean algebras with interior operators by McKinsey and Tarski (1944).

Definition

1. A McKinsey-Tarski algebra or an MT-algebra is a complete Boolean algebra with an interior operator $\Box$ (that is,
$\Box 1 = 1$, $\Box(a \wedge b) = \Box a \wedge \Box b$, $\Box a \leq a$, and $\Box a \leq \Box\Box a$).
2. An MT-morphism between MT-algebras M and N is a complete Boolean homomorphism $h : M \to N$ satisfying $h(\Box_M a) \leq \Box_N h(a)$ for each $a \in M$.
3. Let **MT** be the category of MT-algebras and MT-morphisms.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

This defines a contravariant functor $\mathcal{P}$: **Top** $\to$ **MT**.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

This defines a contravariant functor $\mathcal{P}$: **Top** $\to$ **MT**. The functor at : **MT** $\to$ **Top**, sending an MT-algebra to its space of atoms, is *right adjoint* to $\mathcal{P}$.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

This defines a contravariant functor $\mathcal{P} : \mathbf{Top} \to \mathbf{MT}$. The functor $\mathrm{at} : \mathbf{MT} \to \mathbf{Top}$, sending an MT-algebra to its space of atoms, is *right adjoint* to $\mathcal{P}$.

A spatial MT-algebra (SMT) is an atomic MT-algebra.

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

This defines a contravariant functor $\mathcal{P} : \mathbf{Top} \to \mathbf{MT}$. The functor $\mathrm{at} : \mathbf{MT} \to \mathbf{Top}$, sending an MT-algebra to its space of atoms, is *right adjoint* to $\mathcal{P}$.

A spatial MT-algebra (SMT) is an atomic MT-algebra.

Theorem

The adjuction $(\mathcal{P}, \mathrm{at})$ restricts to a dual equivalence between **SMT** *and* **Top***.*

Top and MT

For each topological space X, $(\mathcal{P}(X), \mathrm{int})$ is an MT-algebra.

If $f : X \to Y$ is a continuous function between topological spaces, then $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ is an MT-morphism.

This defines a contravariant functor $\mathcal{P}$: **Top** $\to$ **MT**. The functor at : **MT** $\to$ **Top**, sending an MT-algebra to its space of atoms, is *right adjoint* to $\mathcal{P}$.

A spatial MT-algebra (SMT) is an atomic MT-algebra.

Theorem

The adjuction $(\mathcal{P}, \mathrm{at})$ *restricts to a dual equivalence between* **SMT** *and* **Top**.

This shows that **MT** is a faithful generalization of **Top**.

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.
2. We define a relation $a \prec b$ provided there is a finite subset T of $\mathrm{LC}(M)$ such that $a \leq \bigvee T \leq b$

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.
2. We define a relation $a \prec b$ provided there is a finite subset T of $\mathrm{LC}(M)$ such that $a \leq \bigvee T \leq b$ (equivalently, $a \prec b$ provided there is x in the Boolean subalgebra of M generated by $\mathcal{O}(M)$ such that $a \leq x \leq b$).

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.
2. We define a relation $a \prec b$ provided there is a finite subset T of $\mathrm{LC}(M)$ such that $a \leq \bigvee T \leq b$ (equivalently, $a \prec b$ provided there is x in the Boolean subalgebra of M generated by $\mathcal{O}(M)$ such that $a \leq x \leq b$).
3. M is a T_D-algebra if $a = \bigvee\{b \in M \mid b \prec a\}$ for all $a \in M$.

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.
2. We define a relation $a \prec b$ provided there is a finite subset T of $\mathrm{LC}(M)$ such that $a \leq \bigvee T \leq b$ (equivalently, $a \prec b$ provided there is x in the Boolean subalgebra of M generated by $\mathcal{O}(M)$ such that $a \leq x \leq b$).
3. M is a T_D-algebra if $a = \bigvee\{b \in M \mid b \prec a\}$ for all $a \in M$.
4. Let $\mathbf{MT}_D$ be the full subcategory of $\mathbf{MT}$ consisting of T_D-algebras, and let $\mathbf{SMT}_D$ be the full subcategory of $\mathbf{MT}_D$ consisting of spatial T_D-algebras.

T_D-algebras

Let $M \in \mathbf{MT}$. An element $a \in M$ is open if $a = \Box a$. Let $\mathcal{O}(M)$ be the collection of open elements of M.

Definition

1. An element $x \in M$ is locally closed if $x = u \wedge \neg v$, where $u, v \in \mathcal{O}(M)$. Let $\mathrm{LC}(M)$ be the set of locally closed elements of M.
2. We define a relation $a \prec b$ provided there is a finite subset T of $\mathrm{LC}(M)$ such that $a \leq \bigvee T \leq b$ (equivalently, $a \prec b$ provided there is x in the Boolean subalgebra of M generated by $\mathcal{O}(M)$ such that $a \leq x \leq b$).
3. M is a T_D-algebra if $a = \bigvee\{b \in M \mid b \prec a\}$ for all $a \in M$.
4. Let $\mathbf{MT}_D$ be the full subcategory of $\mathbf{MT}$ consisting of T_D-algebras, and let $\mathbf{SMT}_D$ be the full subcategory of $\mathbf{MT}_D$ consisting of spatial T_D-algebras.

Theorem

The dual equivalence between **Top** *and* **SMT** *restricts to a dual equivalence between* $\mathbf{Top}_D$ *and* $\mathbf{SMT}_D$.

Comparison

$$\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] \arrow[d, equal] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top} \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathsf{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}$$

Comparison

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] \arrow[d, equal] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top} \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathsf{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top}_D \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT}_D \arrow[u, hook, "full"']
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathsf{pt}_\mathsf{D}"] \\
\mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"']
\end{tikzcd}
$$

Comparison

$$\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] \arrow[d, equal] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top} \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT} \arrow[u, hook, "\mathit{full}"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathsf{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "\mathit{full}"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SFrm} \arrow[u, hook, "\mathit{full}"']
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top}_D \arrow[u, hook, "\mathit{full}"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT}_D \arrow[u, hook, "\mathit{full}"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathsf{pt}_\mathsf{D}"] \\
\mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "\mathit{full}"']
\end{tikzcd}$$

The setting of frames lacks the expressive power to tell the difference between T_D and non T_D-spaces.

Comparison

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] \arrow[d, equal] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top} \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT} \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\Omega"] & \mathbf{Frm} \arrow[l, bend left, "\mathsf{pt}"] \\
\mathbf{Sob} \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SFrm} \arrow[u, hook, "full"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathbf{Top} \arrow[r, bend left, "\mathcal{P}"] & \mathbf{MT} \arrow[l, bend left, "\mathsf{at}"] \\
\mathbf{Top}_D \arrow[u, hook, "full"] \arrow[r, leftrightarrow, "\cong"'] & \mathbf{SMT}_D \arrow[u, hook, "full"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathbf{Top}_D \arrow[r, bend left, "\Omega"] \arrow[d, equal] & \mathbf{Frm}_D \arrow[l, bend left, "\mathsf{pt_D}"] \\
\mathbf{Top}_D \arrow[r, leftrightarrow, "\cong"'] & \mathbf{TD\text{-}SFrm}_D \arrow[u, hook, "full"']
\end{tikzcd}
$$

The setting of frames lacks the expressive power to tell the difference between T_D and non T_D-spaces. We use the richer language of MT-algebras to define the T_D-hull of a frame.

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$.

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$. We lift the interior operator $\Box : \mathcal{B}L \to \mathcal{B}L$ to $\overline{\Box} : \overline{\mathcal{B}L} \to \overline{\mathcal{B}L}$ by

$$\overline{\Box}a = \bigvee \{\Box b \mid b \in \mathcal{B}L \text{ and } b \leq a\}.$$

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$. We lift the interior operator $\Box : \mathcal{B}L \to \mathcal{B}L$ to $\overline{\Box} : \overline{\mathcal{B}L} \to \overline{\mathcal{B}L}$ by

$$\overline{\Box} a = \bigvee \{\Box b \mid b \in \mathcal{B}L \text{ and } b \leq a\}.$$

Then $(\overline{\mathcal{B}L}, \overline{\Box})$ is an MT-algebra such that $\mathcal{O}(\overline{\mathcal{B}L}) \cong L$.

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$. We lift the interior operator $\square : \mathcal{B}L \to \mathcal{B}L$ to $\overline{\square} : \overline{\mathcal{B}L} \to \overline{\mathcal{B}L}$ by

$$\overline{\square} a = \bigvee \{\square b \mid b \in \mathcal{B}L \text{ and } b \leq a\}.$$

Then $\left(\overline{\mathcal{B}L}, \overline{\square}\right)$ is an MT-algebra such that $\mathcal{O}(\overline{\mathcal{B}L}) \cong L$.

Definition

For a frame L, we call the MT-algebra $\left(\overline{\mathcal{B}L}, \overline{\square}\right)$ the Funayama envelope of L and denote it by $\mathcal{F}(L)$.

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$. We lift the interior operator $\Box : \mathcal{B}L \to \mathcal{B}L$ to $\overline{\Box} : \overline{\mathcal{B}L} \to \overline{\mathcal{B}L}$ by

$$\overline{\Box} a = \bigvee \{\Box b \mid b \in \mathcal{B}L \text{ and } b \leq a\}.$$

Then $(\overline{\mathcal{B}L}, \overline{\Box})$ is an MT-algebra such that $\mathcal{O}(\overline{\mathcal{B}L}) \cong L$.

Definition

For a frame L, we call the MT-algebra $(\overline{\mathcal{B}L}, \overline{\Box})$ the Funayama envelope of L and denote it by $\mathcal{F}(L)$.

Theorem

1. *For a frame L, $\mathcal{F}(L)$ is T_D.*

Funayama envelope as the T_D-hull

For a frame L, let $\overline{\mathcal{B}L}$ be the MacNeille completion of its Boolean envelope $\mathcal{B}L$. We lift the interior operator $\Box : \mathcal{B}L \to \mathcal{B}L$ to $\overline{\Box} : \overline{\mathcal{B}L} \to \overline{\mathcal{B}L}$ by

$$\overline{\Box}a = \bigvee \{\Box b \mid b \in \mathcal{B}L \text{ and } b \leq a\}.$$

Then $(\overline{\mathcal{B}L}, \overline{\Box})$ is an MT-algebra such that $\mathcal{O}(\overline{\mathcal{B}L}) \cong L$.

Definition

For a frame L, we call the MT-algebra $(\overline{\mathcal{B}L}, \overline{\Box})$ the Funayama envelope of L and denote it by $\mathcal{F}(L)$.

Theorem

1. *For a frame L, $\mathcal{F}(L)$ is T_D.*
2. *M is T_D iff $\mathcal{F}(\mathcal{O}(M)) \cong M$.*

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

1. $f|_{\mathcal{O}(M)} : \mathcal{O}(M) \to \mathcal{O}(N)$ is a frame homomorphism.

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

1. $f|_{\mathcal{O}(M)} : \mathcal{O}(M) \to \mathcal{O}(N)$ is a frame homomorphism.
2. $f(a \wedge b) = f(a) \wedge f(b)$ for each $a, b \in M$.

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

1. $f|_{\mathcal{O}(M)} : \mathcal{O}(M) \to \mathcal{O}(N)$ is a frame homomorphism.
2. $f(a \wedge b) = f(a) \wedge f(b)$ for each $a, b \in M$.
3. $a_1 \prec b_1$ and $a_2 \prec b_2$ imply $f(a_1 \vee a_2) \prec f(b_1) \vee f(b_2)$.

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

1. $f|_{\mathcal{O}(M)} : \mathcal{O}(M) \to \mathcal{O}(N)$ is a frame homomorphism.
2. $f(a \wedge b) = f(a) \wedge f(b)$ for each $a, b \in M$.
3. $a_1 \prec b_1$ and $a_2 \prec b_2$ imply $f(a_1 \vee a_2) \prec f(b_1) \vee f(b_2)$.
4. $f(a) = \bigvee\{f(b) \mid b \in M,\ b \prec a\}$ for each $a \in M$.

Proximity morphisms

The assignment $L \to \mathcal{F}(L)$ is not functorial: frame morphisms do not in general lift to complete Boolean homomorphisms between their Funayama envelopes.

In order to remedy this, we introduce a weaker notion of morphism:

Definition

For $M, N \in \mathbf{MT}$, a map $f : M \to N$ is a proximity morphism provided

1. $f|_{\mathcal{O}(M)} : \mathcal{O}(M) \to \mathcal{O}(N)$ is a frame homomorphism.
2. $f(a \wedge b) = f(a) \wedge f(b)$ for each $a, b \in M$.
3. $a_1 \prec b_1$ and $a_2 \prec b_2$ imply $f(a_1 \vee a_2) \prec f(b_1) \vee f(b_2)$.
4. $f(a) = \bigvee\{f(b) \mid b \in M,\ b \prec a\}$ for each $a \in M$.

Remark

Proximity morphisms between MT-algebras are reminiscent of de Vries morphisms.

Main Theorem

Let $\mathbf{PMT}_D$ be the category of T_D-algebras with proximity morphisms.

Main Theorem

Let $\mathbf{PMT}_D$ be the category of T_D-algebras with proximity morphisms.

$\mathcal{F}$ extends frame homomorphisms to proximity morphisms, yielding:

Main Theorem

Let **$\mathbf{PMT}_D$** be the category of T_D-algebras with proximity morphisms.

$\mathcal{F}$ extends frame homomorphisms to proximity morphisms, yielding:

Lemma

$\mathcal{F} : \mathbf{Frm} \to \mathbf{PMT}_D$ *is a functor.*

Main Theorem

Let $\mathbf{PMT}_D$ be the category of T_D-algebras with proximity morphisms.

$\mathcal{F}$ extends frame homomorphisms to proximity morphisms, yielding:

Lemma

$\mathcal{F} : \mathbf{Frm} \to \mathbf{PMT}_D$ *is a functor.*

Main Theorem

$\mathbf{Frm}$ *is equivalent to* $\mathbf{PMT}_D$*.*

T_D-coreflection of T_0-spaces

${}_D$**Top** the wide subcategory of **Top** whose morphisms f are such that $\Omega(f)$ is a D-morphism.

T_D-coreflection of T_0-spaces

${}_D\mathbf{Top}$ the wide subcategory of $\mathbf{Top}$ whose morphisms f are such that $\Omega(f)$ is a D-morphism.

${}_D\mathbf{Top}_0$ the full subcategory of ${}_D\mathbf{Top}$ consisting of T_0-spaces.

T_D-coreflection of T_0-spaces

${}_D\mathbf{Top}$ the wide subcategory of $\mathbf{Top}$ whose morphisms f are such that $\Omega(f)$ is a D-morphism.

${}_D\mathbf{Top}_0$ the full subcategory of ${}_D\mathbf{Top}$ consisting of T_0-spaces.

Theorem (Banaschewski-Pultr)

$\mathbf{Top}_D$ *is a coreflective subcategory of* ${}_D\mathbf{Top}_0$.

T_D-coreflection of T_0-spaces

${}_D\mathbf{Top}$ the wide subcategory of **Top** whose morphisms f are such that $\Omega(f)$ is a D-morphism.

${}_D\mathbf{Top}_0$ the full subcategory of ${}_D\mathbf{Top}$ consisting of T_0-spaces.

Theorem (Banaschewski-Pultr)

$\mathbf{Top}_D$ *is a coreflective subcategory of* ${}_D\mathbf{Top}_0$.

Remark

The coreflection can be achieved by the inclusion of locally closed points of a T_0-space X into X.

T_D-coreflection of T_0-spaces

${}_D\mathbf{Top}$ the wide subcategory of $\mathbf{Top}$ whose morphisms f are such that $\Omega(f)$ is a D-morphism.

${}_D\mathbf{Top}_0$ the full subcategory of ${}_D\mathbf{Top}$ consisting of T_0-spaces.

Theorem (Banaschewski-Pultr)

$\mathbf{Top}_D$ *is a coreflective subcategory of* ${}_D\mathbf{Top}_0$.

Remark

The coreflection can be achieved by the inclusion of locally closed points of a T_0-space X into X.

Drawback

While it is easy to descibe the T_D-coreflection in terms of topological spaces, the notion of T_D-reflection is not expressible in the language of frames.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.
2. Let ${}_D\mathbf{MT}$ be the category of MT-algebras and D-morphisms.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.
2. Let ${}_D\mathbf{MT}$ be the category of MT-algebras and D-morphisms. We also let ${}_D\mathbf{SMT}$ be the full subcategory of ${}_D\mathbf{MT}$ consisting of spatial MT-algebras.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.
2. Let ${}_D\mathbf{MT}$ be the category of MT-algebras and D-morphisms. We also let ${}_D\mathbf{SMT}$ be the full subcategory of ${}_D\mathbf{MT}$ consisting of spatial MT-algebras.

Theorem

The category $\mathbf{SMT}_D$ *is a reflective subcategory of* ${}_D\mathbf{SMT}$.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.
2. Let ${}_D\mathbf{MT}$ be the category of MT-algebras and D-morphisms. We also let ${}_D\mathbf{SMT}$ be the full subcategory of ${}_D\mathbf{MT}$ consisting of spatial MT-algebras.

Theorem

The category $\mathbf{SMT}_D$ is a reflective subcategory of ${}_D\mathbf{SMT}$.

Corollary

$\mathbf{Top}_D$ *is a coreflective subcategory of* ${}_D\mathbf{Top}$.

Pointfree description of the T_D-coreflection of spaces.

Definition

1. An MT-morphism f is a D-morphism if the left adjoint f^* maps locally closed atoms to locally closed atoms.
2. Let ${}_D\mathbf{MT}$ be the category of MT-algebras and D-morphisms. We also let ${}_D\mathbf{SMT}$ be the full subcategory of ${}_D\mathbf{MT}$ consisting of spatial MT-algebras.

Theorem

The category $\mathbf{SMT}_D$ is a reflective subcategory of ${}_D\mathbf{SMT}$.

Corollary

$\mathbf{Top}_D$ is a coreflective subcategory of ${}_D\mathbf{Top}$.

Our pointfree approach yields the T_D-coreflection for all spaces, lifting the T_0 restriction of the Banaschewski-Pultr construction.

Thank you!