

Fine grained analysis of conservative Maltsev CSP

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Feder and Vardi famously conjectured in the nineties that a finite domain constraint satisfaction problem whose constraint relations are chosen from some predetermined template is either NP-complete or solvable in polynomial time. Bulatov, Jeavons, and Krokhin suggested a general algebraic criterion for determining the tractability of a finite domain CSP in terms of the polymorphism algebra of its template. The CSP dichotomy conjecture and the stronger algebraic CSP dichotomy conjecture initiated a fertile area of research connecting universal algebra to complexity theory.

While the algebraic dichotomy conjecture was proven true independently by Bulatov and Zhuk in 2017, there are still questions concerning the fine-grained classification of the fixed template CSPs within P, up to logspace reductions. For example, while the famous Bulatov-Dalmau algorithm does show every Maltsev CSP is tractable, it is still an open question whether every Maltsev CSP belongs to the complexity class NC.

By providing a new algorithm, we are able to show that every finite domain fixed template CSP possessing a conservative Maltsev polymorphism belongs to the complexity class Parity L (which is contained in NC), which consists of all problems that can be solved by a Turing machine with a work tape of logarithmic size in the sense that the positive instances of the problem are precisely those where the machine has an even number of accepting computation paths. In fact, conservative Maltsev CSPs are either in L or they are complete for Parity L, so we now have a full complexity classification up to

logspace reductions for conservative Maltsev CSP.
This is joint work with Manuel Bodirsky.