Algebras of conditionals

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In this contribution we introduce and study a logico-algebraic notion of conditional operator. A conditional statement is a hypothetical proposition of the form "If [antecedent] is the case, then [consequent] is the case", where the antecedent is assumed to be true. Such a notion can be formalized by expanding the language of classical logic by a binary operator a/b that reads as "a given b". A most well-known approach in this direction comes from a philosophical perspective developed first by Stalnaker, and further analyzed by Lewis, that in order to axiomatize the operator "/" ground their investigation on particular Kripke-like structures. The novel approach we propose here is grounded in the algebraic setting of Boolean algebras, where we show that there is a natural way of formalizing conditional statements starting from the algebraic notion of "quotient". Given a Boolean algebra B and an element b in B, one can define a new Boolean algebra, say B/b, intuitively obtained by assuming that b is true. More in details, one considers the congruence collapsing b and the truth constant 1, and then B/b is the corresponding quotient. Then the idea is to define a conditional operator "/" such that a/b represents the element a as seen in the quotient B/b, mapped back to B. The particular structural properties of Boolean algebras allow us to do so in a natural way. Translating this intuition using Stone duality, we define a class of standard models and then we analyze the variety QA generated by them. In this work, we menage to axiomatize QA and we prove that it is a subvariety of the variety of Lewis variably strict conditional algebras VA. We then provide an

algebraic study of this variety, which in particular turns out to be a discriminator variety. This fact in particular entails that the classes of subdirectly irreducible, directly indecomposable, and simple algebras in QA coincide, and in this case they are exactly the class of (isomorphic copies of) our standard models.