Decidability and undecidability in substructural logics

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Substructural logics constitute resource-sensitive generalizations of classical and of intuitionistic logic and include linear logic, relevance logic, and many-valued logic; their study goes back to the 1920's. Their algebraic semantics, residuated lattices, have their independent history (starting in the 1930's) and include structures such as lattice-ordered groups, lattices of ideals of rings, Heyting and Boolean algebras, and relation algebras. Moreover, the deductive systems associated with substructural logics connect to linguistics (both applied and mathematical) and they have applications to computer science (for example in pointer management and memory allocation in concurrent programming).

We will survey decidability and undecidability results for substructural logics and also discuss the computational complexity of some of them. The tools for getting undecidability and lower complexity bounds include encoding counter machines. Methods for decidability and for upper bounds include a proof-theoretic analysis of the derivational systems and also the use of well quasi-ordered sets. On the way we will mention rational semantics for these logics and use them both to establish the correctness of the encodings, as well as the extraction of finite countermodels from the proof-theoretic systems.