Varieties of MV-monoids and positive MV-algebras

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We investigate MV-monoids and their subquasivarieties. MV-monoids are algebras $\langle A, \vee, \wedge, \oplus, \odot, 0, 1\rangle$ where $\langle A, \vee, \wedge, 0, 1\rangle$ is a bounded distributive lattice, $\langle A, \oplus, 0\rangle$ and $\langle A, \odot, 1\rangle$ are commutative monoids, and some further connecting axioms are satisfied. Every MV-algebra in the signature $\{\oplus, \neg, 0\}$ is term equivalent to an algebra that has an MV-monoid as a reduct, by defining, as standard, $1 := \neg 0$, $x \odot y := \neg(\neg x \oplus \neg y)$, $x \vee y := (x \odot \neg y) \oplus y$ and $x \wedge y := \neg(\neg x \vee \neg y)$. Particular examples of MV-monoids are positive MV-algebras, i.e. the $\{\vee, \wedge, \oplus, \odot, 0, 1\}$-subreducts of MV-algebras. Positive MV-algebras form a peculiar quasivariety in the sense that, albeit having a logical motivation (being the quasivariety of subreducts of MV-algebras), it is not the equivalent quasivariety semantics of any logic. We study the lattice of subvarieties of MV-monoids and describe the lattice of subvarieties of positive MV-algebras. We characterize the finite subdirectly irreducible positive MV-algebras. Furthermore, we axiomatize all varieties of positive MV-algebras.

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