Pure Embeddings in Acts: Stability and Cofibrant Generation

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Acts — sets with a monoid action — form a natural generalization of modules. In this talk, we study the class of S-acts over a fixed monoid S with pure embeddings. This setting fits into the framework of an abstract elementary class (AEC), a generalization of elementary model theory designed to study classes of structures that are not axiomatizable in first-order logic. In contrast to module theory, where all first-order theories are stable, it is known that some firstorder theories of acts are unstable. This makes the study of stability particularly interesting in this setting. We show that the AEC of S-acts with pure embeddings has a stable independence relation if and only if the monoid S satisfies that, for every $s, t \in S$, either $s \in St$ or $t \in Ss$. We use this result to show that pure embeddings in acts are cofibrantly generated — that all pure monomorphisms can be constructed from a *set* via pushouts, transfinite composition, and retracts.

This is joint work with S. Cox, M. Kamsma, M. Mazari-Armida, J. Rosický.