What are coz-inclusions?

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The framework categories in point-free topology are the category of frames and its dual, the category of locales. The former has an algebraic nature, while the latter has a more geometrical flavor. Together, they provide a richer way to study spaces. Locales are referred to as generalized spaces, and sublocales (or frame quotients) are the generalized versions of subspaces in the category of locales. We will study a special class of sublocales in which one can observe how complex the interaction between the lattice-theoretic part of frames and the topological aspect of sublocales can be.

The cozero part of a frame is a sub-σ-frame inside it that plays an important role under complete regularity, since it join-generates the frame. The relationship between the cozero part of a sublocale and its ambient frame is generally not well-behaved; in fact, it is quite chaotic. Nevertheless, there are several types of embeddings that help control the behavior of the cozero part of a sublocale.

In this talk, we will study the new notion of a *coz-included* sublocale: a special kind of embedding that describes how the cozero part of a sublocale sits with respect to the cozero part of the ambient frame. This new notion will be compared with other well-known forms of embedding, such as C-, C^*-, and z-embeddings. Motivated by non-trivial spatial examples, this new notion and its weakenings—such as casi coz-included and relative zero sublocales—will be explored and characterized. As a consequence, new characterizations of several classes of frames, such as perfectly normal and Oz frames, among others, will be provided.

This is a joint work with Oghenetega Ighedo and Joanne Walters-Wayland.