The E-base of finite semidistributive lattices

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Implicational bases (IBs) are a well-known representation of finite closure spaces and their closure lattices. Implications go by many names in a broad range of fields, e.g., attribute implications in Formal Concept Analysis, functional dependencies in database theory, or Horn clauses in propositional logic.

In lattice theory, implications translate into join covers, i.e., sets of relations $j \leq j_1 \vee \cdots \vee j_k$, $k \geq 1$, on the set of join-irreducible elements of a lattice.

The representation by an IB is not unique, and a closure space usually admits multiple IBs. Among these, the canonical base, the canonical direct base as well as the D-base aroused significant attention due to their structural and algorithmic properties.

The study of free lattices was influential in bringing up an IB of a new sort, which was called the E-base. It is a refinement of the D-base that, unlike the aforementioned IBs, does not always accurately represent its associated closure space. This leads to an intriguing question: for which classes of (closure) lattices do closure spaces have a valid E-base?

Finite lower bounded lattices are known to form such a class. In recent publication `https://arxiv.org/abs/2502.04146`, we prove that for semidistributive lattices, the E-base is both valid and minimum.

Among other results, we look into E-base in a few classes of closures spaces with the Exchange Axiom, establish the complexity of recognizing that two elements of closure space defined by some IB are

E-related, and show that D-geometries—closure spaces corresponding to lower bounded lattices with the Anti-Exchange axiom—can be recognized from any IB in polynomial time, using the help from the E-base.

This is a joint work with Simon Vilmin, Aix-Marseille Université, CNRS, France.