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Lattices
Universal Algebra
Boolean Algebras
Set Theory
Topology

ABSTRACTS

University of Colorado Boulder

Contents

Simple truncated Archimedean vector lattices

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Certain features of an archimedean vector lattice A can be usefully analyzed by means of a *truncation operation*, an operation $a \mapsto \overline{a}$ on the positive cone A^+ satisfying three simple axioms. An archimedean vector lattice endowed with a truncation, or *trunc* for short, can always be represented as a family of continuous extended-real valued functions on a compact Hausdorff pointed space which are finite on a dense subset and which vanish at the designated point; when so represented, we denote the elements $\tilde{a}$, $\tilde{b}$, etc.

For example, the truncation operation makes it easy to pick out those elements which behave like step functions.

Lemma. *The following are equivalent for an element $u \geq 0$ in a trunc A.*

(1) *u is a unital component of A, i.e., $u \wedge v$ is a component of v for any $v \in \overline{A} \equiv \{\, \overline{a} : a \in A^+ \,\}$.*
(This means $(u \wedge v) \wedge (v - u \wedge v) = 0$).

(2) *$u = \overline{2u}$.*

(3) *$\tilde{u}$ is the characteristic function χ_U of a clopen subset $U \subseteq X$ omitting the designated point $*$, i.e., $\tilde{u}(x) = 1$ for $x \in U$ and $\tilde{u}(x) = 0$ for $x \notin U$.*

The set of unital components of any trunc A, designated $\mathcal{UC}(A)$, forms a nonempty generalized Boolean algebra, i.e., a lattice with least element 0 which admits relative complementation: for all $a, b \in A$ there exists $c \in A$ such that $c \vee b = a \vee b$ and $c \wedge b = \bot$. An element a in a trunc A is called *simple* if it is a linear combination of unital components. The trunc is called simple if all its elements are simple. In any trunc A, the set of simple elements forms a subtrunc σA, and this is the biggest simple subtrunc of A.

Proposition. *The category* **sT** *of simple truncs is equivalent to the category* **gBa** *of generalized Boolean algebras, which is equivalent to the category* $\mathbf{BSp}_*$ *of pointed Boolean spaces.*

An element a of a trunc A is called *bounded* if it satisfies $|a| \leq n\overline{|a|}$ for some n. The trunc is called bounded if all its elements are bounded. In any trunc A, the set of bounded elements forms a subtrunc A^*, and this is the biggest bounded subtrunc of A.

An element $a \geq 0$ of a trunc A is called *bounded away from* 0 if there is some $\varepsilon > 0$ such that $\tilde{g}(x) > \varepsilon$ for all x such that $\tilde{g}(x) > 0$. It can be show that this is equivalent to $\overline{na} \in \mathcal{UC}(A)$ for some positive integer n. The trunc A is said to be *bounded away from* 0 if each $a \in A^+$ is bounded away from 0.

Theorem. *The following are equivalent for a trunc A.*

(1) *A is simple.*

(2) *A is (isomorphic to) the family $\mathcal{LC}X$ of locally constant functions which vanish at the designated point of a compact Hausdorff pointed space X.*

(3) *A is bounded and bounded away from* 0*.*

(4) *A is hyperarchimedean and has enough unital components, i.e., for all $a \in A^+$ there exists $u \in \mathcal{UC}(A)$ such that $\overline{a} \leq u$.*

(5) *A is hyperarchimedean and bounded away from ∞, i.e., each $a \in A^+$ has the feature that $\tilde{a}$ vanishes on a neighborhood of the designated point $*$.*

The Complexity of Homomorphism Factorization

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We investigate the computational complexity of the problem of deciding if an algebra homomorphism can be factored through an intermediate algebra. Specifically, we fix an algebraic language, $\mathcal{L}$, and take as input an algebra homomorphism $f\colon X \to Z$ between two finite $\mathcal{L}$-algebras X and Z, along with an intermediate finite $\mathcal{L}$-algebra Y. The decision problem asks whether there are homomorphisms $g\colon X \to Y$ and $h\colon Y \to Z$ such that $f = hg$. We show that this problem is NP-complete for most languages. We also investigate special cases where homomorphism factorization can be performed in polynomial time.

A non-pointfree approach to pointfree topology

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A frame or locale is a complete lattice satisfying the join infinite distributive law. Frames are the primary subject of study in pointfree topology. The lattice of opens of a topological space is a typical example of a frame. Such frames are known as spatial frames. But not every frame is spatial. Such frames are usually studied using algebraic means as the standard topological machinery may no longer be available. The aim of this tutorial is to present a different approach to study frames through their spectra of prime filters. This makes the machinery of Priestley and Esakia dualities available and restores some of the lost geometric intuition.

Tutorial I: Basics of pointfree topology.
Tutorial II: Basics of Priestley and Esakia dualities.
Tutorial III: The study of frames through their spectra of prime filters.

Seven questions about ω^*

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The space $\omega^* = \beta\omega \setminus \omega$ is the Stone-Čech remainder of the countable discrete space ω, and is the Stone dual of the Boolean algebra $\mathcal{P}(\omega)/\mathrm{fin}$. In this talk I will survey seven open questions about ω^*, four of them old and three of them new. In the process I will explain some of my recent work on the role of ω^* in topological dynamics, and how the dynamical structures living on ω^* shed light on some of these old, classic questions from set-theoretic topology.

Defining n Choose k Algebras to Generate $\mathbb{M}_m$ representers

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The finite lattice representation problem asks whether every finite lattice is isomorphic to the congruence lattice of some finite algebra. A class of lattices for which this conjecture is unresolved is the $\mathbb{M}_m$ lattices for $m \in \mathbb{N}$, the lattices with a minimum element Δ, a maximum element ∇, and m pairwise incomparable elements in between. The purpose of this talk is to discuss the possible structures of transitive G-set representers of $\mathbb{M}_m$ lattices, and describe techniques which could be used to generate new representers of $\mathbb{M}_m$ lattices. Our primary result states that if $\operatorname{Con}(A, G) \cong \mathbb{M}_m$ for $m \geq 3$, then (A, G) falls under one of two possible cases: Either $|A/\theta| = \sqrt{|A|}$ for all $\theta \in \operatorname{Con}(A, G) \setminus \{\Delta, \nabla\}$, or (A, G) is what we call an n choose k algebra for some $n \geq 3$ and $2 \leq k \leq n-1$. An n choose k algebra is constructed from a set $\underline{A}$ of n columns, a group $\underline{G}$ of permutations acting transitively on $\underline{A}$, and a collection $\overline{A}$ of rows, where a row is a k-element set of columns. Then $A = \{(\underline{x}, \overline{x}) \in \underline{A} \times \overline{A} : \underline{x} \in \overline{x}\}$, and each $f \in G$ corresponds to a $\underline{f} \in \underline{G}$ such that $f(\underline{x}, \overline{x}) = (\underline{f}(\underline{x}), \underline{f}[\overline{x}])$. We discuss methods for generating n choose k algebras, either from a transitive group and a single row, or as a factor algebra of a different G-set. Finally, we explicitly give the congruence lattices for the n choose k algebras with the symmetric and alternating groups.

Stone duality for infinitary logic

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The classical Stone duality allows a Boolean algebra to be canonically recovered from its space of ultrafilters, as well as providing a topological characterization of the class of all such spaces. For a finitary propositional theory, Stone duality allows the syntax of the theory (its Lindenbaum–Tarski algebra) to be recovered from its semantics (its space of models): the usual completeness and definability theorems for propositional logic. Many known variants and generalizations of Stone duality have analogous interpretations as completeness–definability theorems for various fragments of finitary propositional and first-order logic.

We will present such a duality theorem for the countably infinitary first-order logic $\mathcal{L}_{\omega_1\omega}$: the syntax of every countable $\mathcal{L}_{\omega_1\omega}$-theory, in the form of its category of imaginary sorts and definable functions, can be canonically recovered from its standard Borel groupoid of countable models. Furthermore, the class of standard Borel groupoids arising in this manner can be characterized by a non-Archimedean Polishability condition.

Descriptive Classification of Abelian Orbit Equivalence Relations

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We discuss some recent results and conjectures regarding the classification under Borel reduction of the orbit equivalence relations which are induced by Borel actions of abelian Polish and standard Borel groups.

A generalization of affine algebras

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An algebra is said to be affine if it is polynomially equivalent to a module over a ring. By a theorem of Herrmann, the class of affine algebras turns out to be exactly equivalent to the class of abelian Mal'cev algebras. In the hopes of better understanding nilpotent Mal'cev algebras, I define a class of algebras that generalizes the class of affine algebras. After proving some basic results, I will focus on algebras whose cardinality is the product of two distinct primes. I will end with an open question on finite nilpotent loops.

On extensions of partial isometries

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In this paper we define a notion of S-extension for a metric space and study minimality and coherence of S-extensions. We give a complete characterization of all finite minimal S-extensions of a given finite metric space. We also define a notion of ultraextensive metric spaces and show that every countable metric space can be extended to a countable ultraextensive metric space. As an application, we show that every countable subset of the Urysohn metric space can be extended to a countable dense ultraextensive subset of the Urysohn space. We also study compact ultrametric spaces and show that every compact ultrametric space can be extended to a compact ultraextensive ultrametric space. Finally, we study a similar problem for general compact metric spaces and give an equivalent formulation for a positive solution to the problem.

Spectra of commutative BCK-algebras

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In 1966, Imai and Iséki introduced BCK-algebras as the algebraic counterparts of implication-reducts of classical (or non-classical) propositional logics. The class of all BCK-algebras is a quasivariety, but the class of *commutative* BCK-algebras, which will be the focus of this talk, form a variety. Following well-known constructions for commutative rings, Boolean algebras, and distributive lattices, we topologize the set of prime ideals to obtain a space referred to as the **spectrum** of a commutative BCK-algebra. In this talk we will discuss what is known about the spectrum and what is not, as well as some recent results and even more recent open questions, with a particular focus on a nice family of examples.

The complexity of the equation solvability and equivalence problems over finite groups

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We investigate the complexity of the equation solvability and the equivalence problems for finite groups. The equivalence problem over a finite group $\mathbf{G}$ asks whether two expressions p and q over $\mathbf{G}$ are equivalent or not. The equation solvability problem over a finite group $\mathbf{G}$ asks whether or not an equation $p = q$ has a solution over $\mathbf{G}$.

By previous results, these problems can be solved in polynomial time for nilpotent groups. Moreover, for non-solvable groups the equation solvability is NP-complete and the equivalence is coNP-complete. The complexity of these problems for solvable, non-nilpotent groups are only known for a few meta-Abelian groups. The smallest group for which these complexities were not known before is $\mathbf{SL}(2, \mathbb{Z}_3)$. We determine the complexity of the equivalence and equation solvability problem for groups of the form $\mathbf{P} \rtimes \mathbf{A}$, where $\mathbf{P}$ is a p-group and $\mathbf{A}$ is an Abelian group. For such groups these problems turn out to be decidable in polynomial time.

Generalized bunched-implication logics

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Bunched-implication logic is a substructural logic where both conjunction and strong conjunction have corresponding implications (and therefore both distribute over disjunction). BI logic has applications to the theory of pointers in computer science and is related to separation logic. The algebraic semantics, known as BI-algebras, are Heyting algebras that further support an additional commutative residuated lattice structure. We discuss BI-algebras and their non-commutative generalizations and in particular we characterize their congruences.

Computational Complexity of Matrix Semigroup Properties

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For a matrix semigroup $S = \langle a_1, \ldots, a_k \rangle \leq \mathbb{F}^{n\times n}$, we show that the following can be performed in polynomial time: (1) enumerate the left and right identities of S, and (2) determine if S is nilpotent. Also, as an update for last year's presentation, we will briefly summarize recent results for transformation semigroups.

On the structure of idempotent residuated lattices

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The results reported in this talk are joint work with José Gil-Ferez and George Metcalfe (University of Bern). We provide descriptions of the totally ordered members of residuated lattices that are idempotent as monoids and obtain counting theorems for the number of finite algebras in various subclasses. We also establish the finite embeddability property for any variety generated by a class defined by positive universal formulas in the language $\{\vee, \cdot, e\}$ and including the formula $\forall x, y\,(xy = x \text{ or } xy = y)$. We then make use of a more symmetric version of James Rafterys characterization theorem for totally ordered commutative idempotent residuated lattices to prove that the variety generated by this class has the amalgamation property. We address an open problem in the literature by giving an example of a noncommutative variety of idempotent residuated lattices that has the amalgamation property.

We also give an example of a noncommutative idempotent involutive residuated lattice. In joint work with Olim Tuyt (University of Bern) and Diego Valota (University of Milan), we prove that commutative idempotent involutive residuated lattices are unions of Boolean algebras under the order $x \sqsubseteq y \iff xy = x$. All members up to size 16 are commutative, and we conjecture that this holds for all finite members of this variety.

Quasibands

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Nonassociative idempotent magmas arise naturally in various settings such as the faces of a building or chains in modular lattices. In this talk I will describe a variety of magmas we call *quasibands*, which arise as (sub)reducts of bands (idempotent semigroups): in a band $(B, \cdot)$, define a new operation $\circ$ by $x \circ y = xyx$. This is analogous to how quandles arise as subreducts of groups under the conjugation operations. The quasiband operation $\circ$ is sometimes used as a notational shorthand (especially in the theory of noncommutative lattices) or to characterize band properties. For instance, $(B, \circ)$ is associative, hence a left regular band, if and only if $(B, \cdot)$ is a regular band.

The variety of quasibands is defined by 4 identities. Our main result is that this variety is precisely the variety of $\circ$-subreducts of bands. In addition, I will talk about the natural preorder and natural partial order on a quasiband, the center of a quasiband, and the relationship between free quasibands and free bands.

This is joint work with Tomaž Pisanski (Ljubljana).

The equivalence problem for nilpotent algebras

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The *(circuit) equivalence problem* $\mathsf{CEQV}(\mathbf{A})$ of a finite algebra $\mathbf{A}$ is the computational problem where the input consists of two polynomials $p(x_1, \ldots, x_n)$ and $q(x_1, \ldots, x_n)$ (encoded by circuits), and the task is to decide whether $\mathbf{A} \models p(x_1, \ldots, x_n) \approx q(x_1, \ldots, x_n)$ or not.

Although the complexity is widely open in general, for algebras $\mathbf{A}$ from congruence modular varieties much is known: If $\mathbf{A}$ is supernilpotent, $\mathsf{CEQV}(\mathbf{A})$ is in P by a result of Aichinger and Mudrinski. Idziak and Krzaczkowski showed that every non-nilpotent $\mathbf{A}$ has a homomorphic image $\mathbf{A}'$, such that $\mathsf{CEQV}(\mathbf{A}')$ is coNP-complete. This essentially only leaves a gap in the nilpotent, non-supernilpotent case.

In this talk I would like to discuss very recent results, showing that then $\mathsf{CEQV}(\mathbf{A})$ is in P. This is part of ongoing work with Jacek Krzaczkowski and Piotr Kawałek.

Relation algebras of Sugihara, Belnap, Meyer, Church. Relevance logic of Tarski

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Sugihara's relation algebra is a proper relation algebra containing chains of relations isomorphic to Sugihara's original matrices. Belnap's matrices form a definitional reduct of a proper relation algebra known as the Point Algebra. The 2-element and 4-element Sugihara matrices are definitional subreducts of the Point Algebra. Meyer's crystal matrices, Meyer's RM84 matrices, and Church's diamond matrices are also definitional subreducts of proper relation algebras. The representation of Sugihara matrices as algebras of binary relations, together with Meyer's characterization of R-mingle by Sugihara matrices, yields the characterization of R-mingle by sets of transitive dense relations that commute under composition. The axioms of the Dunn-McCall system R-mingle can therefore be understood as statements about binary relations.

The definition of Tarski's relevance logic arises from his work on first-order logic restricted to finitely many variables. It is inspired by the characterization of equations true in all relation algebras as those provable in first-order logic restricted to 4 variables. It has Belnap's variable-sharing property and avoids the paradoxes of implication. It contains the Basic Logic of Routley, Plumwood, Meyer, Brady, along with several derived rules of inference. It does not include several formulas used as axioms in the Anderson Belnap system R, such as Contraposition. It contains a formula (outside both R and R-mingle) that is a counterexample to a completeness theorem of Kowalski (that the system R is complete with respect to the class of dense commutative relation algebras).

Congruence 5-permutability is not join prime

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Let $\mathbb{P} = (P; \leq)$ be the 6-element poset with the order $0 \leq a, b \leq c, d \leq 1$. This poset has a compatible 5-ary near-unanimity operation and thus its clone $\operatorname{Pol}\mathbb{P}$ of polymorphisms is finitely generated by $p_1, \ldots, p_k \in \operatorname{Pol}\mathbb{P}$. The algebra $\mathbf{P} = (P; p_1, \ldots, p_k)$ generates a finitely based congruence distributive variety $\mathcal{P}$. Let $\mathcal{M}$ be the variety of algebras with a single ternary basic operation m satisfying the majority identities. Neither $\mathcal{P}$, nor $\mathcal{M}$ is congruence n-permutable for any n, however we show that their varietal coproduct, the class of algebras having basic operations $p_1, \ldots, p_k, m$ and satisfying the defining identities of both $\mathcal{P}$ and $\mathcal{M}$, is congruence 5-permutable. This means, that the strong Maltsev condition $\mathcal{D}_5$ of having Hagemann–Mitschke terms for congruence 5-permutability is not join prime in the lattice of interpretability types.

Finitely Decidable Varieties

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All varieties mentioned are locally finite. A satisfactory characterization of varieties whose first order theory is decidable, or recursive (the decidable varieties) exists. Every such variety is the varietal product of three varieties, $\mathcal{V} = \mathcal{S} \otimes \mathcal{A} \otimes \mathcal{D}$, where $\mathcal{D}$ is a discriminator variety, $\mathcal{A}$ is an affine variety, and $\mathcal{S}$ is a strongly Abelian variety derived from a category consisting of functions between some objects and satisfying a linear divisibility condition. Given such a decomposition of a finitely generated variety $\mathcal{V}$, the variety $\mathcal{V}$ is decidable iff $\mathcal{A}$ is decidable. $\mathcal{A}$ is determined by a finite ring with unit and is decidable iff the variety of left unitary modules over that ring is decidable. (A characterization of the finite rings with this property is still lacking.)

A variety if said to be finitely decidable iff the first order theory of the class of finite members of $\mathcal{V}$ is decidable. No characterization of the finitely decidable varieties is known, although for congruence modular varieties, P. Idziak obtained a good structural characterization of the finitely decidable varieties. J. Jeong found examples of varieties that are decidable and finitely undecidable, also finitely decidable but undecidable.

In this lecture, I will talk about the history of research on decidable varieties, and describe many conditions, on congruences, that are necessary if finite decidability is to hold.

Infinite games for teams

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Given a game of length ω where I and II take turns playing sequences of fixed length τ, we may reinterpret player I as a team of τ players I(t) for $t < \tau$ (and likewise reinterpret II). Call a winning strategy for team I *independent* if, for each t, the plays of I(t) depend only on previous plays of II(t). Using this concept we obtain new characterizations of the cardinals $\aleph_n$ for $n < \omega$. Call a winning strategy for team I *semi-independent* if, for each t, the plays of I(t) depend only on previous plays of II(u) for $u \leq t$. Using this concept we obtain Fubini-type theorems for some topological games related to measure and category.

Finite degree clones are undecidable

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A clone is a set of operations on a set that is closed under composition and variable manipulations. There are two common methods of finitely specifying a clone of operations. The first is to generate the clone from a finite set of operations via composition and variable manipulations. The second method is to specify the clone as all operations preserving a given finite set of relations. Clones specified in the first way are called finitely generated, and clones specified in the second way are called finitely related.

Since the 1970s, it has been conjectured that there is no algorithmic procedure for deciding whether a finitely generated clone is finitely related. We give an affirmative answer to this conjecture by associating to each Minsky machine $\mathcal{M}$ a finitely generated clone $\mathcal{C}$ such that $\mathcal{C}$ is finitely related if and only if $\mathcal{M}$ halts, thus proving that deciding whether a given clone is finitely related is impossible.

On interpretations between propositional logics

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Propositional logics can be endowed with a notion of *interpretability* that generalizes a classical concept from universal algebra. When ordered under the interpretability relation, the collection of all propositional logics forms a poset whose underlying universe is a proper class. Part of the interest of this poset is that its set-complete filters can be identified with classes of logics that play a fundamental role in abstract algebraic logic. Since varieties of algebras can be identified with suitable propositional logics, these filters encompass Maltsev classes from universal algebras.

In this talk (based on joint work with R. Jansana) we will explore the structure of the poset of all logics ordered under interpretability, and the decomposability of some of its set-complete filters.

Profinite completions of MV-algebras

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In the first part of the talk, we compute the profinite completion of a general MV-algebra. We obtain that the profinite completion of an MV-algebra is the direct product of all its finite simple homomorphic images. As immediate byproducts of our description, we obtain simpler proofs of some previously known results such as the profinite completion of a Boolean algebra, the action of profinite completions on the Boolean center of regular MV-algebras, the characterization of MV-algebras that are isomorphic to their own profinite completions. We also deduce the functoriality of the profinite completion defined on **MV**.

In the second part of the talk, we use the description of the profinite completion found to characterize profinite MV-algebras that are isomorphic to profinite completions of some MV-algebras. In particular, we prove that a profinite MV-algebra $A := \prod_{x\in X} Ł_{n_x}$ is isomorphic to the profinite completion of an MV-algebra if and only if there exists a compact Hausdorff space Y containing X as a dense subspace and a separating subalgebra A' of $\mathrm{Cont}(Y)$ satisfying:
(i) For every $x \in X$, J_x has rank n_x in A', where $J_x := \{f \in A' : f(x) = 0\}$; and (ii) For every $y \in Y \setminus X$, J_y has infinite rank in A'.

We use this to obtain several classes of profinite MV-algebras that are isomorphic to the profinite completion of other MV-algebras.

Algebras of fractions

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A classical result of early abstract algebra, essentially due to Steinitz (1910), states that each cancellative commutative semigroup can be embedded into an Abelian group. This result can be extended beyond the commutative case: according to a theorem of Ore (1931), each so-called right reversible cancellative semigroup can be embedded into a group of right fractions, i.e. a group where each element has the form $a \cdot b^{-1}$ for some a, b in the original semigroup.

In this talk, we will show how these classical results can be subsumed under a more general construction of ordered algebras of fractions, which will also allow us to construct algebras of fractions of e.g. Heyting algebras. Such algebras of fractions will not necessarily be groups, but rather they will lie in wider class of ordered algebras called *involutive residuated posets*. Examples of such structures are Boolean algebras, MV-algebras, Sugihara monoids, relation algebras, and of course partially ordered groups (of which ordinary groups are a special case).

In order to formulate this general construction, we introduce new algebraic structures called *bimonoids* and show that they form an appropriate framework for investigating algebras of fractions. As a by-product we obtain some new categorical equivalences between categories of residuated structures, as well as a uniform proof of some known ones due to Bahls et al. (2003), Galatos & Raftery (2014), and Fussner & Galatos (2017).

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On the number of clonoids

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A clonoid is a set of finitary functions from a set A to a set B that is closed under taking minors. Hence clonoids are generalizations of clones. Clonoids have recently been investigated in connection to classifying the complexity of Promise Constraint Satisfaction Problems. By a classical result of Post, there are only countably many clones on a 2-element set. In contrast to that, we present continuum many clonoids for $A = B = \{0, 1\}$. More generally, for any finite set A and any 2-element algebra $\mathbf{B}$, we give the cardinality of the set of clonoids from A to $\mathbf{B}$ that are closed under the operations of $\mathbf{B}$.

Subvariety containment for idempotent semirings

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Residuated lattices are well known algebraic structures that also have an important connection to mathematical logic, as they are the equivalent algebraic semantics of substructural logics. Residuated lattices (RL) have an idempotent semiring (ISR) reduct with the signature $\{\vee, \cdot, 1\}$. The subvarieties of RL axiomatized by particular ISR-equations, namely those that are *simple equations*, correspond to extensions of the Full Lambek calculus (**FL**) by *simple structural rules*. Such extensions were shown to enjoy cut-admissibility by Galatos and Jipsen (2013), and have garnered much attention specifically with respect to decidability. For example, Horčík (2015) proved that subvarieties of RL that contain a specific algebra have an undecidable word problem. As a consequence of these works, in order to study decidability it is important to understand the subvariety containment for subvarieties of RL axiomatized by ISR-equations. We prove that, interestingly, this can be studied exactly in the ISR setting. More precisely, for a set $\Sigma \cup \{\epsilon\}$ of simple ISR-equations, $\mathsf{RL} + \Sigma \models \epsilon$ if and only if $\mathsf{ISR} + \Sigma \models \epsilon$. This correspondence is achieved using a residuated frames construction, which in many cases also provides a decision procedure for the $\{\vee, \cdot, 1\}$-fragment of the equation theory.

Gluing residuated lattices

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Residuated lattices constitute the equivalent algebraic semantics of substructural logics, which encompass most of the interesting non-classical logics: intuitionistic logic, fuzzy logics, relevance logics, linear logic, besides including classical logic as a limit case. Thus, the investigation of the variety of residuated lattices is a powerful tool for analyzing such logics comparatively. The multitude of different structures makes the study fairly complicated, and at the present moment large classes of residuated lattices lack a structural understanding. Thus, the study of constructions that allow to obtain new structures starting from known ones, is extremely important to improve our grasp of residuated lattices, and as a result of substructural logics.

We introduce and study new constructions that *glue* together pairs of integral residuated lattices, that share either a principal filter, or a principal filter and a principal ideal. The former construction generalizes the *ordinal sum* construction first introduced by Ferreirim, that has played an important role in the study of residuated structures. Intuitively, an ordinal sum is a gluing, where we only glue together the top elements of the lattices.

Gaps between cardinalities of quotient algebras of rank-into-rank embeddings

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Suppose that $j_1, \ldots, j_r : V_\lambda \to V_\lambda$ are non-trivial elementary embeddings. Then whenever $\gamma < \lambda$, the cardinality $|\langle j_1, \ldots, j_r\rangle/\equiv^\gamma|$ is finite, and $\{\mathrm{crit}(j) \mid \langle j_1, \ldots, j_r\rangle\}$ has order type ω. Let $\mathrm{crit}_n(j_1, \ldots, j_r)$ denote the n-th element in the set $\{\mathrm{crit}(j) \mid \langle j_1, \ldots, j_r\rangle\}$, and let $X_n = \langle j_1, \ldots, j_r\rangle/\equiv^{\mathrm{crit}_n(j_1,\ldots,j_r)}$ for $n \in \omega$. We obtain a sequence of polynomials $(p_{n,j_1,\ldots,j_r}(x_1, \ldots, x_r))_{n\in\omega}$ that satisfies the infinite product formula

$$\prod_{n=0}^{\infty} p_{n,j_1,\ldots,j_r}(x_1, \ldots, x_r) = \frac{1}{1 - (x_1 + \cdots + x_r)}.$$

We shall use these polynomials and some facts from analysis and analytic number theory to produce upper bounds on the cardinalities

$$|\{n \in \omega : |X_{n+1}| - |X_n| = h\}|.$$

Expressivity in some many-valued modal logics

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Modal logic is one of the most developed and studied non-classical logics, yielding a beautiful equilibrium between complexity and expressivity. On the other hand, substructural (and as a particular case, many-valued) logics provide a formal framework to manage vague and resource sensitive information in a very general (and so, adaptable) fashion. *Many-valued modal logics*, combining the notions of modal operators with logics over richer algebraic structures is a field in ongoing development. While the first publications on the topic can be traced back to some seminal works by Fitting in the 90s, it has been only in the latter years when a more systematic work has been done.

In this talk we present some recent results for these logics, focused on their decidability and axiomatizability, and compare their behaviour to classical modal logics and the corresponding propositional substructural logics. In particular, we exhibit a family of undecidable non-classical (global) modal logics, including two of the three better known fuzzy logics (namely modal expansions of Łukasiewicz and Product logics). Moreover, we will see how we can further exploit undecidability to show that these logics are not even R.E., thus not being axiomatizable in the usual sense. This contrasts to what happens in their propositional counterparts, and places then possibly nearer to the behavior of the corresponding FO logics (e.g. validity in FO $[0,1]_{Ł}$ is not R.E. either, [3]).

If time allows, we will also exhibit the different (and better behaved) case of Gödel–Dumett modal logics (the third main fuzzy logic), where finite axiomatic systems and decidability for the minimal cases have been proved. We will see an interesting expressivity condition on this logic, namely that the $\Box$-fragment of the logic arising from the Gödel algebra with universe $\{0\} \cup \{1/i \colon i \in \mathbb{N}^*\}$ differs from the one over $[0,1]_G$ and can be finitely axiomatized. This poses another point of contrast with both the propositional case (where this logic coincides with the usual Gödel logic over $[0,1]_G$) and with the FO case (whose $\forall$- fragment over the above algebra is non arithmetical [1],[2]).

References

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On the Descending Central Series of Higher Commutators in Simple Algebras

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This talk will characterize the potential behaviors of higher commutators in simple algebras.

The Constraint Satisfaction Dichotomy Theorem for beginners

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Given a relational structure $\mathbb{A}$ in a finite signature, the *Constraint Satisfaction Problem over* $\mathbb{A}$, denoted CSP($\mathbb{A}$), is the computational problem of deciding whether a conjunction of atomic formulas in the signature of $\mathbb{A}$ has a solution when interpreted in $\mathbb{A}$. The computational complexity of CSP($\mathbb{A}$) depends on $\mathbb{A}$, and can even be noncomputable if $\mathbb{A}$ is infinite. In the 1990s, T. Feder and M. Vardi conjectured that if $\mathbb{A}$ is finite, then CSP($\mathbb{A}$) is either NP-complete or in P. This *CSP Dichotomy Conjecture* has animated theoretical computer scientists, partnering with universal algebraists, for over 20 years. Finally in 2017, A. Bulatov and D. Zhuk independently posted proofs of the CSP Dichotomy Conjecture on the arXiv.

In this series of three tutorial lectures, I will first introduce the audience to the Dichotomy Conjecture and describe how algebra is relevant. Next I will show through examples how linear equations can be hidden in CSP instances via critical rectangular relations. Finally, I will make precise the following core principle driving Zhuk's proof: if CSP($\mathbb{A}$) is not NP-complete and an instance of CSP($\mathbb{A}$) is locally consistent but has no solution, then the inconsistency is necessarily due to hidden linear equations.

Correspondence, Canonicity, and Model Theory for Monotonic Modal Logics

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We investigate the role of coalgebraic predicate logic, a logic for neighborhood frames first proposed by Chang, in the study of monotonic modal logics. We prove analogues of the Goldblatt-Thomason Theorem and Fine's Canonicity Theorem for classes of monotonic neighborhood frames closed under elementary equivalence in coalgebraic predicate logic. The elementary equivalence here can be relativized to the classes of monotonic, quasi-filter, augmented quasi-filter, filter, or augmented filter neighborhood frames, respectively. The original, Kripke-semantic versions of the theorems follow as a special case concerning the classes of augmented filter neighborhood frames.

Reductions on equivalence relations generated by universal sets

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Let Y be a Polish space, and let $\Gamma \subseteq \wp(Y)$ be given. For a Polish space X, for a relation $A \subseteq X \times Y$, and for $x \in X$, define the x-section of A to be $A_x = \{y \in Y \mid (x, y) \in A\}$. We say that a relation $A \subseteq X \times Y$ is *universal* for Γ if $\Gamma = \{A_x \mid x \in X\}$.

The kernel of the map $x \mapsto A_x$ is an equivalence relation E_A, called the *equivalence relation on X generated by A*. Explicitly, $x \; E_A \; x'$ if and only if $A_x = A_{x'}$. In this talk I will discuss the complexity of equivalence relations generated by universal relations for Polish spaces. This is joint work with Longyun Ding.