

Preface

This volume contains the papers presented at BLAST 2018, held on August 6-10, 2018 at the University of Denver.

July 28, 2018
Denver

Natasha Dobrinen and Nick Galatos.

Table of Contents

Operations that preserve integrability, and truncated Riesz spaces	1
<i>Marco Abbadini</i>	
Jonsson Filters	2
<i>Shehzad Ahmed</i>	
Pointfree pointwise convergence, Baire functions, and epicompletions of truncated archimedean l-groups.....	3
<i>Richard Ball</i>	
Unique amenability of topological groups	4
<i>Dana Bartosova</i>	
The Complexity of Homomorphism Factorization	5
<i>Kevin Berg</i>	
Universal automorphisms of $P(\omega)/\text{fin}$	6
<i>Will Brian</i>	
Bounded commutative residuated lattices with a retraction term.....	7
<i>Manuela Busaniche</i>	
From Lattice-Ordered Groups to Residuated Lattices: Hamiltonian and Nilpotent Varieties	8
<i>Almudena Colacito</i>	
Nilpotent Mal'cev clones on sets of square-free cardinality	9
<i>Eran Crockett</i>	
Completions of Pseudo-ordered Sets.....	10
<i>Maria Cruz</i>	
Restricted kinds of density and the associated ideals	11
<i>Pratulananda Das, Marek Balcerzak, Malgorzata Filipzak and Jaroslav Swaczyna</i>	
On the complexity of computing difference terms and operations	12
<i>William De Meo, Ralph Freese and Matthew Valeriote</i>	
The logic of comparative cardinality.....	13
<i>Yifeng Ding</i>	
Perfect tree forcings for singular cardinals	14
<i>Natasha Dobrinen, Daniel Hathaway and Karel Prikry</i>	
Higher order versions of the logic of chains	15
<i>Mirna Džamonja</i>	
Some peculiar properties of the variety of bounded commutative BCK-algebras	16
<i>Matt Evans</i>	
Variations on a theme: Hindman's theorem.....	17
<i>David Fernandez-Breton</i>	

Autolog Equality Modulation	18
<i>John Fisher</i>	
Distributivity in residuated structures	20
<i>Wesley Fussner and Peter Jipsen</i>	
A duality-theoretic approach to MTL-algebras	21
<i>Wesley Fussner and Sara Ugolini</i>	
Twist products of bimonoids	22
<i>Nick Galatos and Adam Přenosil</i>	
The Tukey Order in Topology and Analysis	23
<i>Paul Gartside</i>	
The super tree property at the successor of a singular cardinal	24
<i>Sherwood Hachtman and Dima Sinapova</i>	
Boolean subalgebras of orthomodular structures	25
<i>John Harding, Chris Heunen, Bert Lindenhovius and Mirko Navara</i>	
Strong Types for Direct Logic	26
<i>Carl Hewitt</i>	
Choice-free Stone duality	28
<i>Wesley Holliday</i>	
Computational complexity of checking various semigroup properties	29
<i>Trevor Jack and Peter Mayr</i>	
The sum of observables on a σ -distributive lattice effect algebra	30
<i>Jiri Janda and Yongming Li</i>	
Group representation, Hahn-type embedding, and the FSSC for $IUL^{\wedge}fp$	32
<i>Sándor Jenei</i>	
Convex effect algebras and the Kalmbach monad	33
<i>Gejza Jenča</i>	
Nonassociative right hoops	34
<i>Peter Jipsen and Michael Kinyon</i>	
A constructive view of weak topologies on a topos	35
<i>Zeinab Khanjanzadehseresti and Ali Madanshekaf</i>	
The logic of Sigma formulas	37
<i>Andre Kornell</i>	
Positive Group Homomorphisms of Free Unital Abelian l-groups	38
<i>Tomas Kroupa</i>	
Some Aspects of Weak Ideal Topology on Acts over Monoids	39
<i>Ali Madanshekaf</i>	
On the universal-algebraic theory of measure and integration	40
<i>Vincenzo Marra</i>	

Homeomorphisms of Cech-Stone remainders: the zero-dimensional case	41
<i>Paul Mckenney and Ilijas Farah</i>	
Uniform Interpolation	42
<i>George Metcalfe</i>	
Between Tukey equivalence and Boolean automorphism	43
<i>David Milovich</i>	
Higher Dimensional Congruences and Supernilpotence	44
<i>Andrew Moorhead</i>	
The role of Bitopology in Natural Duality	45
<i>M. Andrew Moshier</i>	
Parts of biframe and d-frames	47
<i>M. Andrew Moshier, Imanol Mozo Carollo and Joanne Walters-Wayland</i>	
Ramsey spaces and the Katetov order	48
<i>Sonia Navarro Flores</i>	
On injective constructions of S-semigroups	49
<i>Jan Paseka and Xia Zhang</i>	
Distributivity of a segmentation lattice	50
<i>Jan Pavlik</i>	
Terms in the variety generated by N5	51
<i>John Snow</i>	
Promise Constraint Satisfaction Problems	52
<i>Athena Sparks</i>	
Closure operators on the clone lattice defined by ultrapowers	53
<i>Agnes Szendrei and Keith Kearnes</i>	
Hypernatural numbers in ultra-Ramsey theory	54
<i>Timothy Trujillo</i>	
Boolean-valued point-free topology	55
<i>Joseph Van Name</i>	
Uncountable Rainbow Ramsey Theory Revisited	56
<i>Jing Zhang</i>	

OPERATIONS THAT PRESERVE INTEGRABILITY, AND TRUNCATED RIESZ SPACES

MARCO ABBADINI

For $(\Omega, \mathcal{F}, \mu)$ a measure space, we set $\mathcal{L}^1(\mu) := \{f: \Omega \rightarrow \mathbb{R} \mid f \text{ is } \mathcal{F}\text{-measurable and } \int_{\Omega} |f| d\mu < \infty\}$. The aim of this contribution is to determine and axiomatize the equational algebraic structure of such \mathcal{L}^1 spaces. It is well known that $\mathcal{L}^1(\mu)$ is closed under pointwise sum but, in general, it is not closed under pointwise product. Given a function $\tau: \mathbb{R}^I \rightarrow \mathbb{R}$, we show that $\mathcal{L}^1(\mu)$ is closed under τ for every measure μ iff, roughly speaking, τ is measurable and at most linear. In such case, we say that τ *preserves integrability*.

Let \mathcal{V} be the infinitary variety whose operations are the functions $\mathbb{R}^I \rightarrow \mathbb{R}$ (with $I \in \mathbb{N} \cup \{\mathbb{N}\}$) that preserve integrability, and whose axioms are the equations satisfied by \mathbb{R} . Main examples of objects in \mathcal{V} are the \mathcal{L}^1 spaces. We show that \mathcal{V} is isomorphic to the category of Dedekind σ -complete truncated Riesz spaces, where “truncated” is intended in the sense of R. Ball [1]. As primitive operations one can take those of Riesz spaces, Ball’s truncation, along with a truncated version of countable joins. A simple axiomatization of this variety is given. The free object on a set I in \mathcal{V} is shown to be $Free_I := \{\tau: \mathbb{R}^I \rightarrow \mathbb{R} \mid \tau \text{ preserves integrability}\}$.

Analogous results are obtained for the spaces $\mathcal{L}^1(\mu)$, with μ finite. In particular, we show that the corresponding infinitary variety is isomorphic to the category of Dedekind σ -complete Riesz spaces with weak unit. (These results are part of ongoing work towards a Ph.D. degree under the supervision of V. Marra.)

REFERENCES

- [1] Richard N. Ball. Truncated abelian lattice-ordered groups I: The pointed (Yosida) representation. *Topology Appl.*, 162:43–65, 2014.

JÓNSSON FILTERS

SHEHZAD AHMED

ABSTRACT. We say that a cardinal λ is Jonsson if any coloring of the finite subsets of λ in λ -many colors has a weakly homogeneous set. Like most Ramsey-type cardinals, Jonsson cardinals sit between 0^\sharp and a measurable with respect to consistency strength. Unlike most large cardinals, we do not know whether or not it is consistent that there is a singular cardinal with Jonsson successor. If there is such a Jonsson cardinal, then there is a filter which captures these colorings witnessing Jonssonness. In this talk, we use this property of capturing colorings to isolate the notion of Jonsson filters and discuss a number of their properties. Along the way, we use their pcf theory to make precise the link between club guessing and scales.

POINTFREE POINTWISE CONVERGENCE, BAIRE FUNCTIONS, AND EPIMORPHISMS IN TRUNCATED ARCHIMEDEAN ℓ -GROUPS

RICHARD N. BALL

ABSTRACT. We define pointfree pointwise convergence, and use it to define the Baire functions on a locale. The main result is that the Baire functions on a locale coincide with the continuous functions on its P -locale coreflection. Furthermore, we show that the Baire functions on a locale constitute the epicompletion of the continuous functions in the relevant category.

The relevant category is \mathbf{T} , the category of truncated archimedean ℓ -groups, hereafter nicknamed trunks. \mathbf{T} is closely related to the famous category \mathbf{W} of unital archimedean ℓ -groups. The universal objects in \mathbf{T} are of the form \mathcal{R}_0L , the trunc of real-valued locale maps $L \rightarrow \mathbb{R}$ which vanish at the designated point of a pointed locale L .

We provide an intuitive definition of pointwise convergence in \mathcal{R}_0L which extends the classical definition, and show that it has a number of nice properties: all homomorphisms and operations of \mathbf{T} are pointwise continuous, and a pointwise dense extension is a trunc epimorphism. Conversely, we show that every epic extension $G \rightarrow H$ has an epic extension $H \rightarrow K$ such that G is pointwise dense in K .

We show that the rich theory of epimorphisms in \mathbf{W} carries over to \mathbf{T} with only minor modification. In particular, the epicomplete trunks comprise a full monoreflective subcategory, and are characterized as those objects of the form \mathcal{R}_0P for a P -locale P . In light of these facts, a reformulation of the last clause of the preceding paragraph is that any trunc is pointwise dense in any epicompletion. And a trunc is epicomplete iff it is pointwise complete, i.e., has no proper extension in which it is pointwise dense.

Finally, for a given pointed locale L , we define the functions of Baire class α on L in the classical fashion. A function is Baire class 0 if it lies in \mathcal{R}_0L , and of Baire class β if it is the pointwise limit of a sequence of functions of Baire class $\alpha < \beta$. A Baire function on L is a function of Baire class α for some α . Our results can be summarized as follows.

Theorem. *For a pointed locale L with P -locale coreflection $\mathcal{P}_*L \rightarrow L$, the Baire functions on L are precisely the continuous functions on \mathcal{P}_*L , i.e., those of $\mathcal{R}_0\mathcal{P}_*L$.*

Theorem. *The embedding $\mathcal{R}_0L \rightarrow \mathcal{R}_0\mathcal{P}_*L$ is the functorial epicompletion in \mathbf{T} .*

UNIQUE AMENABILITY OF TOPOLOGICAL GROUPS

DANA BARTOŠOVÁ

A topological group G is called amenable if whenever G acts on a compact Hausdorff space X there is a measure on X that is invariant under the action. An amenable group is uniquely amenable if every action on a compact Hausdorff space with a dense orbit admits a unique invariant measure. Precompact groups are trivially uniquely amenable and we show that in many classes of groups these are the only examples.

THE COMPLEXITY OF HOMOMORPHISM FACTORIZATION

KEVIN M. BERG

ABSTRACT. We investigate the computational complexity of the problem of deciding if an algebra homomorphism can be factored through an intermediate algebra. Specifically, we fix an algebraic language, \mathcal{L} , and take as input an algebra homomorphism $f: X \rightarrow Z$ between \mathcal{L} -algebras X and Z , along with an intermediate \mathcal{L} -algebra Y . The decision problem asks whether there are homomorphisms $g: X \rightarrow Y$ and $h: Y \rightarrow Z$ such that $f = hg$. We present arguments that these homomorphism factorization problems have NP-complete instances for finite semi-groups. We also develop a notion of homomorphism compatible restriction and show that homomorphism factorization problems have polynomial time instances for finite Boolean algebras, finite vector spaces, finite G -sets, and finite abelian groups.

(Kevin M. Berg) DEPARTMENT OF MATHEMATICS, UNIVERSITY OF COLORADO, BOULDER, CO
80309-0395, U.S.A.

Email address: Kevin.Berg@Colorado.EDU

Speaker: Will Brian

Title: Universal automorphisms of $\mathcal{P}(\omega)/\text{fin}$

Abstract:

Given two automorphisms φ and ψ of $\mathcal{P}(\omega)/\text{fin}$, we say that φ *embeds in* ψ when there is a self-embedding $e : \mathcal{P}(\omega)/\text{fin} \rightarrow \mathcal{P}(\omega)/\text{fin}$ such that $e \circ \varphi = \psi \circ e$.

$$\begin{array}{ccc} \mathcal{P}(\omega)/\text{fin} & \xrightarrow{\psi} & \mathcal{P}(\omega)/\text{fin} \\ \uparrow e & & \uparrow e \\ \mathcal{P}(\omega)/\text{fin} & \xrightarrow{\varphi} & \mathcal{P}(\omega)/\text{fin} \end{array}$$

Equivalently, φ embeds in ψ if there is a subalgebra \mathbb{A} of $\mathcal{P}(\omega)/\text{fin}$ such that $(\mathcal{P}(\omega)/\text{fin}, \varphi)$ is isomorphic to $(\mathbb{A}, \psi \upharpoonright \mathbb{A})$. An automorphism of $\mathcal{P}(\omega)/\text{fin}$ is called *universal* if every other automorphism of $\mathcal{P}(\omega)/\text{fin}$ embeds in it.

The main result I wish to discuss is that, assuming **CH**, there are $2^{2^{\aleph_0}}$ universal automorphisms of $\mathcal{P}(\omega)/\text{fin}$, and some of these are even trivial automorphisms (i.e., automorphisms induced by a function $\omega \rightarrow \omega$). I will also discuss the open question of whether **OCA + MA** implies there is a universal automorphism of $\mathcal{P}(\omega)/\text{fin}$. So far, it is known that there is a “jointly universal” pair of two automorphisms, meaning that every automorphism of $\mathcal{P}(\omega)/\text{fin}$ embeds in at least one of the two.

Bounded commutative residuated lattices with a retraction term.

MANUELA BUSANICHE

Based on ideas developed with Roberto Cignoli, Miguel Andrés Marcos and Sara Ugolini.

Substructural logics encompass many of the interesting nonclassical logics: intuitionistic logic, fuzzy logic, relevance logic, linear logic, besides including classical logic as a limit case. Residuated lattices are the algebraic semantics of substructural logics, that is why their investigation is one of the main tools to understand and study those logical systems uniformly. But the multitude of different structures makes the study fairly complicated, thus the investigation of interesting subvarieties of residuated lattices is an appealing problem to address.

In this talk we study subvarieties of bounded commutative residuated lattices with a retraction term. Many authors have already studied the case when the image of the retraction is a Boolean algebra. Now we investigate the structure of algebras with a retraction onto a hyperarchimedean MV-algebra, which includes the Boolean case. We introduce the notion of *generalized rotation* of a residuated lattice, and after showing that they have an MV-retraction, we characterize the varieties these generalized rotations generate. These varieties include among others: Product algebras, Stonean residuated lattices, BL_n -algebras, perfect and bipartite MTL-algebras, Nilpotent Minimum algebras, Regular Nelson residuated lattices, SBP_0 -algebras.

The purpose is to use the image and the kernel of the retraction term to represent each algebra by simpler and better-known structures: we present a categorical equivalence between our varieties generated by generalized rotations and categories whose objects are triples formed by two algebras and a connecting map. The categorical equivalence helps us to understand the structure of these novel algebras.

Our construction has a dual aim: on one hand, it provides a common framework to compare classes of algebras treated independently in previous studies. On the other hand, it allows us to obtain particular results for important well-known classes of algebras and also new varieties that are certainly worth further investigation.

From Lattice-Ordered Groups to Residuated Lattices: Hamiltonian and Nilpotent Varieties

Almudena Colacito

University of Bern, Switzerland

Nilpotent lattice-ordered groups (nilpotent ℓ -groups, for short) of class $c \in \mathbb{N}$ are those ℓ -groups with shortest central series of length at most c , and form a variety that can be defined relative to the variety of ℓ -groups by a semigroup equation (see, e.g., [4, 6]). Hamiltonian ℓ -groups are those for which every convex subalgebra is normal. The connection between these two classes goes beyond the fact that they can be seen as generalizations of the variety of Abelian ℓ -groups. It was shown in [3] that every nilpotent ℓ -group is Hamiltonian (cf. [8]), and a negative answer to the question whether the variety generated by all nilpotent ℓ -groups is the largest Hamiltonian variety was given in [1].

In this joint work with Davide Fazio and Constantine Tsinakis, we exploit well-known results about nilpotent and Hamiltonian ℓ -groups to argue for similar results about nilpotent and Hamiltonian prelinear cancellative residuated lattices. More precisely, we start by giving a positive answer to the question whether there exists a largest variety of Hamiltonian prelinear cancellative residuated lattices. Later, we extend work done in [5], and provide a categorical equivalence between nilpotent cancellative residuated lattices and nilpotent ℓ -groups endowed with a conucleus. By means of this result, nilpotent prelinear cancellative residuated lattices are proved to be semilinear and Hamiltonian. Finally, relying on results from [7, 2], we prove the failure of the amalgamation property for the variety of nilpotent class- c prelinear cancellative residuated lattices.

- [1] V. V. Bludov and A. M. W. Glass. On the variety generated by all nilpotent lattice-ordered groups. *Transactions of the American Mathematical Society*, 5179–5192, 2006.
- [2] J. Gil-Férez, A. Ledda and C. Tsinakis. The failure of the amalgamation property for semilinear varieties of residuated lattices. *Mathematica Slovaca*, 65(4):817–828, 2015.
- [3] V. M. Kopytov. Lattice-ordered locally nilpotent groups. *Algebra and Logic*, 14(4):249–251, 1975.
- [4] A. I. Malcev. Nilpotent semigroups. *Uchen. Zap. Ivanovsk. Ped. Inst.*, 4(1953):107–111.
- [5] F. Montagna and C. Tsinakis. Ordered groups with a conucleus. *Journal of Pure and Applied Algebra*, 214(1):71–88, 2010.
- [6] B. H. Neumann and T. Taylor. Subsemigroups of nilpotent groups. *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences*, 274(1356):1–4, 1963.
- [7] W. B. Powell and C. Tsinakis. Amalgamations of lattice-ordered groups. *Lattice-Ordered Groups: Advances and Techniques*, A. M. W. Glass and W. C. Holland, eds., 33(3):308–327, 1989.
- [8] N. R. Reilly. Nilpotent, weakly abelian and Hamiltonian lattice ordered groups. *Czechoslovak Mathematical Journal*, 33(3):348–353, 1983.

For each positive square-free integer m that is a product of c distinct primes, we construct $c!$ different nilpotent Mal'cev clones on a set of cardinality m . The constructed clones are nilpotent of class c and dualizable.

More generally, we ask the following questions: is every nilpotent Mal'cev clone on a set of square-free cardinality dualizable? Are each of these clones finitely-generated? How many such clones are there on a given set? What is the relational degree of these clones? These questions all have answers when the clone is abelian. We attempt to generalize to non-abelian nilpotent clones.

“Completions of Pseudo-ordered sets”

Maria Cruz Quiñones

A pseudo-ordered set (X,R) is a set X with a binary relation R that is reflexive and anti-symmetric. If the relation R is additionally transitive, we say it is a partially ordered set. Examples of pseudo-ordered sets include tournaments.

In 1971 Skala proved that every pseudo-ordered set can be embedded into a complete pseudo-ordered set, that is, into a pseudo-ordered set where each subset has a greatest lower bound and a least upper bound. However, Skala’s completion is complex, and lacks many desirable properties. For example, when applied to a complete partially ordered set it can produce a result that is not partially ordered.

We give a new completion that is similar to the usual completion by cuts used to produce the extended reals from the rationals.

RESTRICTED KINDS OF DENSITY AND THE ASSOCIATED IDEALS

MAREK BALCERZAK, PRATULANANDA DAS, MALGORZATA FILIPCZAK, AND JAROSŁAW SWACZYNA

ABSTRACT. We consider modified notions of natural density of subsets of $\omega := \{0, 1, \dots\}$. Namely, we introduce the density of the weight $g: \omega \rightarrow [0, \infty)$ where $g(n) \rightarrow \infty$ and $n/g(n) \rightarrow 0$. The aim of this paper is to study possible consequences of this idea. We study the associated ideals \mathcal{Z}_g of sets $A \subset \omega$ with g -density zero. We show that \mathcal{Z}_g is an $F_{\sigma\delta}$ P-ideal on ω which is generated by a lower semicontinuous submeasure on ω . We find sufficient conditions for proper inclusions of type $\mathcal{Z}_{g_1} \subsetneq \mathcal{Z}_{g_2}$. Also, we show examples where no inclusion between \mathcal{Z}_{g_1} and \mathcal{Z}_{g_2} holds.

INSTITUTE OF MATHEMATICS, ŁÓDŹ UNIVERSITY OF TECHNOLOGY, WÓLCZAŃSKA 215, 93-005 ŁÓDŹ, POLAND
E-mail address: `marek.balcerzak@p.lodz.pl`

DEPARTMENT OF MATHEMATICS, JADAVPUR UNIVERSITY, KALKATA-700032, INDIA
E-mail address: `pratulananda@yahoo.co.in`

FACULTY OF MATHEMATICS AND COMPUTER SCIENCES, ŁÓDŹ UNIVERSITY, UL. BANACHA 22, 90-938 ŁÓDŹ,
POLAND
E-mail address: `malfil@math.uni.lodz.pl`

INSTITUTE OF MATHEMATICS, ŁÓDŹ UNIVERSITY OF TECHNOLOGY, WÓLCZAŃSKA 215, 93-005 ŁÓDŹ, POLAND
E-mail address: `jswaczyna@wp.pl`

TITLE

On the complexity of computing difference terms and operations

KEYWORDS

idempotent equational class, difference term, Malcev condition, computational complexity

ABSTRACT

At BLAST 2017 we presented a positive answer to the following practical question: given a finite idempotent algebra A , can we efficiently decide whether the variety $V(A)$ has a difference term? In this talk we present a further practical result related to this problem. Specifically, we discuss the complexity of constructing a difference term d for $V(A)$ and show that, although d may have exponential length, nonetheless there is a polynomial-time algorithm that takes as input any algebra B in $V(A)$ and produces the operation table for d interpreted in B .

AUTHORS

Name	Country	Institution	URL
William DeMeo	United States	University of Colorado, Boulder	http://williamdemeo.org
Ralph Freese	United States	University of Hawaii, Manoa	http://math.hawaii.edu/~ralph
Matthew Valeriote	Canada	McMaster University	https://ms.mcmaster.ca/~matt

SUBMITTED Jun 01, 05:31 GMT

THE LOGIC OF COMPARATIVE CARDINALITY

YIFENG DING

For any field of sets, there is a natural binary relation on it: aRb if and only if the cardinality of a is at least as large as that of b . Just as the laws of intersection, union, and complementation of sets in a field of sets are captured by the laws of Boolean algebras, we ask what are the laws one must add when we also want to compare cardinalities of sets. Specifically, we consider the (quantifier-free) language of Boolean algebras with an extra binary predicate $|s| \geq |t|$ which is interpreted on fields of sets, with $|s| \geq |t|$ expressing that there is an injection from set t to set s . Then the question of what laws to add becomes how to completely axiomatize the valid formulas in this language.

Previously, axiomatizations of formulas that are valid only for finite sets or only for infinite sets are given, though under interpretations involving not cardinality but probability or possibility. We will bridge the divide here by showing that we can almost define finiteness and infiniteness in this language, and then treat sets differently according to which defining formulas they satisfy. Another feature of our system is that it avoids the complex finite cancellation axioms. Instead, we use de Finetti's axiom together with the so-called polarizability rule, which intuitively states that to prove a formula, we can always choose a set and assume that it is divided into two parts of the same cardinality.

This is a joint work with Matthew Harrison-Trainer and Wesley Holliday.

PERFECT TREE FORCINGS FOR SINGULAR CARDINALS

NATASHA DOBRINEN, DANIEL HATHAWAY, AND KAREL PRIKRY

ABSTRACT. We present some forcing properties of perfect tree forcings defined by Prikry to answer a question of Solovay in the late 1960's regarding first failures of distributivity in complete Boolean algebras. Given a strictly increasing sequence of regular cardinals, where $\langle \kappa_n : n < \omega \rangle$, Prikry defined the forcing \mathbb{P} of all perfect subtrees of $\prod_{n < \omega} \kappa_n$, and proved that for $\kappa = \sup_{n < \omega} \kappa_n$, assuming the necessary cardinal arithmetic, the Boolean completion \mathbb{B} of \mathbb{P} is (ω, μ) -distributive for all $\mu < \kappa$, but (ω, κ, δ) -distributivity fails for all $\delta < \kappa$, implying failure of the (ω, κ) -d.l. These hitherto unpublished results set the stage for the following recent results. \mathbb{P} satisfies a Sacks-type property, implying that \mathbb{B} is $(\omega, \infty, < \kappa)$ -distributive. The $(\mathfrak{h}, 2)$ -d.l. and the $(\mathfrak{d}, \infty, < \kappa)$ -d.l. fail in \mathbb{B} . $\mathcal{P}(\omega)/\text{Fin}$ completely embeds into \mathbb{B} . Also, \mathbb{B} collapses κ^ω to \mathfrak{h} . We further prove that if κ is a limit of countably many measurable cardinals, then \mathbb{B} adds a minimal degree of constructibility for new ω -sequences. Some of these results generalize to cardinals κ with uncountable cofinality.

UNIVERSITY OF DENVER, DEPARTMENT OF MATHEMATICS, C.M. KNUDSON HALL, ROOM 300, 2390 S. YORK ST., DENVER, CO 80208

UNIVERSITY OF DENVER, DEPARTMENT OF MATHEMATICS, C.M. KNUDSON HALL, ROOM 300, 2390 S. YORK ST., DENVER, CO 80208

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MINNESOTA, 127 VINCENT HALL, 206 CHURCH ST. SE, MINNEAPOLIS, MN 55455

Higher order versions of the logic of chains

Mirna Džamonja, School of Mathematics, University of East Anglia, Norwich, UK

Associated Member, IHPST, Université Panthéon-Sorbonne (Paris 1), France

In the spirit of the BLAST conference, we shall discuss a topic that bridges many areas of research in discrete mathematics and mathematical logic. First order logic of chains was discovered by Carol Karp and revisited in recent work of Dž. with Jouko Väänänen. The results have shown that the logic, defined through a singular cardinal of countable cofinality, behaves very much like the first order logic. In our new joint work, we study higher order versions of the logic of chains and their fragments to defend the thesis that in this context we can also recover similarities with the ordinary logic. We also discuss the idea of infinite computation

Some peculiar properties of the variety of bounded commutative BCK-algebras

In this talk I will introduce the variety of bounded commutative BCK-algebras and discuss some its properties. For example, it is arithmetical, not semisimple, not locally finite, and not a discriminator variety. However, it has an infinite family of discriminator subvarieties. Similarly, while the variety does not admit a natural duality, it has an infinite family of subvarieties which do.

Variations on a theme: Hindman's theorem

David José Fernández-Bretón

Keywords: Ramsey theory, Hindman's theorem, Infinitary combinatorics

Hindman's theorem is a Ramsey-theoretic result asserting that, whenever one colours the set of natural numbers with finitely many colours, there will be an infinite set such that all numbers that can be obtained by adding finitely many elements from the set (no repetitions allowed) have the same colour. A significant part of my research work has revolved around exploring generalizations and extensions of this theorem: replacing "natural numbers" with "abelian group" and varying the number of colours, as well as the size of the desired monochromatic set (attempting to obtain uncountable monochromatic sets yields very strong negative results, whereas increasing the number of colours to infinitely many yields some partial positive results, as long as one only attempts to obtain finite monochromatic sets); and more recently, exploring this general theme in the context of set theory without the Axiom of Choice. In this talk I will provide an overview of these results, which have been obtained over the course of the past two years.

Autolog Equality Modulation
jrfisher@cpp.edu
ABSTRACT

A new logic programming language (and theorem prover) design is currently being developed, called Autolog. The abstract computation model for Autolog is based on a Skolem Machine for coherent logic (<http://SkolemMachines.org>). Previous implementations of Skolem Machines have not fully embraced direct modulating actions for equality reasoning combined with coherent logic inferencing. It is planned to augment Autolog with appropriate renditions of equality modulation in order to enable Prover9-like capabilities for Autolog.

The BLAST presentation will explain some fundamental equality modulation design issues for Autolog:

- (1) proving algebraic logic theorems via Autolog,
- (2) implementing parallel equality modulation via concurrency programming,
- (3) using substructural logic equalities as "lemmas" to enable coherent inferences for meta-logics implemented in Autolog.

The design issues will be illustrated with several examples.

In particular, here is an Autolog program/problem modulated by a Heyting algebra equality lemma:

```
// equality lemma
lemma => ¬(A∨B)=¬A∧¬B. // Heyting transform ‡
// coherent logic rules
true => ¬(p(a) ∨ q(a)). // #1
P∧Q => P, Q. // #2
¬q(Z) => goal. // #3
```

Here is an Autolog PROOF tree for the goal...

```
      true
      |
      | via #1
      |
      |-(p(a)∨q(a))
      |
      |-(p(a)∧¬q(a))
      |
      |-(p(a))
      |
      | ...
      |-(q(a))
      |
      | via #3 (Z=a)
      |
      goal
```

‡Proof terms can be algebraic expressions modulated by equality lemmas on the "side".

The "bigger plan" here is something like general mechanisms for "universal logic computations", implemented using a Skolem Machine model.

Distributivity in residuated structures

Wesley Fussner¹ and Peter Jipsen²

¹ University of Denver, Denver, USA
wesley.fussner@du.edu

² Chapman University, Orange, USA
jipsen@chapman.edu

A *residuated binar* is a lattice \mathbf{A} expanded by binary operations $\cdot, \backslash, /$ such that the residuation property

$$y \leq x \backslash z \iff x \cdot y \leq z \iff x \leq z / y$$

holds for all $x, y, z \in A$. Residuated binars satisfy a number of distributive laws connecting the operations $\cdot, \backslash, /$ with the lattice operations (see, e.g., [2]). In particular, residuated binars satisfy the distributive laws

$$\begin{aligned} x(y \vee z) &= xy \vee xz & (x \vee y)z &= xz \vee yz \\ x \backslash (y \wedge z) &= x \backslash y \wedge x \backslash z & (x \wedge y) / z &= x / z \wedge y / z \\ x / (y \vee z) &= x / y \wedge x / z & (x \vee y) \backslash z &= x \backslash z \wedge y \backslash z \end{aligned}$$

but generally do not satisfy lattice distributivity or any of the equations

$$\begin{aligned} x(y \wedge z) &= xy \wedge xz & (x \wedge y)z &= xz \wedge yz \\ x \backslash (y \vee z) &= x \backslash y \vee x \backslash z & (x \vee y) / z &= x / z \vee y / z \\ (x \wedge y) \backslash z &= x \backslash z \vee y \backslash z & x / (y \wedge z) &= x / y \vee x / z \end{aligned}$$

Connections between the six non-trivial distributive laws are known in the context of prelinear residuated lattices [1, 3], but not much is known about the general picture. Here we provide a complete description of implications among the six non-trivial distributive laws in the presence of lattice distributivity, and sketch some additional results in the context of residuated binars with multiplicative neutral element.

References

- [1] K. Blount and C. Tsınakis. The structure of residuated lattices. *Internat. J. Algebra Comput.*, 13(4):437–461, 2003.
- [2] N. Galatos, P. Jipsen, T. Kowalski, and H. Ono. *Residuated Lattices: An Algebraic Glimpse at Substructural Logics*. Elsevier, 2007.
- [3] J.B. Hart, L. Rafter, and C. Tsınakis. The structure of commutative residuated lattices. *Internat. J. Algebra Comput.*, 12(4):509–524, 2002.

A duality-theoretic approach to MTL-algebras

Wesley Fussner and Sara Ugolini

¹ Department of Mathematics, University of Denver
`wesley.fussner@du.edu`

² Department of Computer Science, University of Pisa
`sara.ugolini@di.unipi.it`

In lattice theory, triples constructions date back to Chen and Grätzer’s 1969 decomposition theorem for Stone algebras: each Stone algebra is characterized by the triple consisting of its lattice of complemented elements, its lattice of dense elements, and a homomorphism associating these structures. There is a long history of dual analogues of this construction, with Priestley providing a conceptually-similar treatment on duals in 1974 [6] and Pogel also exploring dual triples in his 1998 thesis [5]. Later on, triples decompositions have been extended to account for richer algebraic structures. For example, [4, 1] have provided similar triples decompositions for classes of MTL-algebras, the algebraic semantics for monoidal t-norm based logic, while the works [2, 3] develop analogous triples representations for classes of residuated lattices. Our aim is to provide a duality theoretic perspective on these recent innovations, showing that the Stone-Priestley duality offers a clarifying framework that sheds light on these constructions.

Our dualized construction gives a uniform way of building the extended Priestley spaces of a large class of MTL-algebras from the Stone spaces of their Boolean skeletons, the extended Priestley spaces of their radicals, and a family of nuclei connecting the two. In order to make this dualized construction possible, we present novel results regarding the extended Priestley duals of MTL-algebras, in particular emphasizing their structure as Priestley spaces enriched by a partial ternary operation. Among other things, this also yields a novel duality for generalized MTL-algebras, in which the aforementioned partial operation is total.

References

- [1] S. Aguzzoli, T. Flaminio, S. Ugolini, *Equivalences between subcategories of MTL-algebras via Boolean algebras and prelinear semihoops*, Journal of Logic and Computation, 27(8): 2525–2549, 2017.
- [2] M. Busaniche, R. Cignoli, M. Marcos, *Stonean residuated lattices*, manuscript.
- [3] M. Busaniche, M. Marcos, S. Ugolini, *Representation by triples of algebras with an MV-retract*, manuscript.
- [4] F. Montagna, S. Ugolini, *A categorical equivalence for product algebras*, Studia Logica, 103(2): 345–373, 2015.
- [5] A. J. Pogel, *Stone triples and self-duality*, PhD Thesis, 1998.
- [6] H. A. Priestley, *Stone lattices, a topological approach*, Fund. Math. 84, 127143, 1974.

Twist products of bimonoids

Nick Galatos¹ and Adam Přenosil²

¹ University of Denver, Denver, Colorado, U.S.A.
`ngalatos@du.edu`

² Institute of Computer Science, Czech Academy of Sciences, Prague, Czechia
`adam.prenosil@gmail.com`

Ordered algebraic structures, especially those arising in non-classical logic, can sometimes be embedded into a sort of canonical complemented envelope: a distributive lattice embeds into its Boolean extension, while a commutative order-cancellative monoid embeds into an ordered Abelian group. We provide a construction which unifies these examples. In particular, we show that each ordered or lattice-ordered bimonoid embeds in a canonical way into a complemented extension, which is an involutive residuated lattice.

Under certain circumstances this construction reduces to a twist product of the original (residuated) bimonoid. This is the case with Brouwerian algebras, whose complemented extensions are what we call strongly idempotent involutive residuated lattices. In fact, each strongly idempotent involutive residuated lattice can be reconstructed in this way from its negative cone, resulting in a categorical equivalence between the variety of Brouwerian algebras and the variety of strongly idempotent involutive residuated lattices.

The Tukey Order in Topology and Analysis

Paul Gartside

2018

Many objects in topology and analysis have a natural order which makes them a directed set: the neighborhood filter of a point, the collection of all compact subsets of a space, and spaces of sequences and functions ordered pointwise. In this tutorial we investigate the properties of these directed sets, and the connections between them.

The main tool is the Tukey order on directed sets, which allows us to compare them ‘up to cofinality’. An important role will be played by chain conditions, in particular calibre (ω_1, ω) .

The super tree property at the successor of a singular cardinal

Sherwood Hachtman

Department of Mathematics, Statistics, and Computer Science
University of Illinois at Chicago

July 31, 2018

Tree properties are a family of combinatorial principles that characterize large cardinal properties, but can consistently hold for successor cardinals. For example, we have the strong and super tree properties, introduced respectively by Jech and Magidor in the 1970's, who showed: If a cardinal is inaccessible, then it has the strong (respectively, super) tree property if and only if it is strongly (super-) compact.

In 1995, Magidor and Shelah showed that if κ is the singular limit of supercompact cardinals, then κ^+ has the tree property. Neeman showed how to force from this situation to obtain the tree property up to $\aleph_{\omega+1}$. In 2011, Fontanella generalized these results to the strong tree property. We discuss how to prove the super tree property holds at a successor of a singular cardinal, and to force the super tree property at $\aleph_{\omega+1}$.

BOOLEAN SUBALGEBRAS OF ORTHOMODULAR STRUCTURES

JOHN HARDING, CHRIS HEUNEN, BERT LINDENHOVIUS, AND MIRKO NAVARA

ABSTRACT. We develop a direct method to recover an orthomodular poset from its poset of Boolean subalgebras. For this a new notion of direction is introduced. Directions are also used to characterize in purely order-theoretic terms those posets that are isomorphic to the poset of Boolean subalgebras of an orthomodular poset. These posets are characterized by simple conditions defining orthodomains and the additional requirement of having enough directions. Excepting pathologies involving maximal Boolean subalgebras of four elements, it is shown that there is an equivalence between the category of orthomodular posets and a suitable category of orthodomains with enough directions with morphisms suitably defined. Furthermore, we develop a representation of orthodomains with enough directions, and hence of orthomodular posets, as certain hypergraphs. This hypergraph approach extends the technique of Greechie diagrams and resembles projective geometry. Using such hypergraphs, every orthomodular poset can be represented by a set of points and lines where each line contains exactly three points.

Strong Types for Direct Logic

Carl Hewitt

<https://plus.google.com/+CarlHewitt-StandardIoT>

In a strongly typed mathematical theory, every proposition, term, and expression has a type where there is no universal type λy . Types are constructed bottom up from primitive types using recursively defined parameterized types. Strong types are extremely important because they block all known paradoxes including Berry, Burali-Forti, Girard, Russell, etc. Blocking known paradoxes makes mathematical theories safer for use in Intelligent Systems by preventing security holes. Consistent strong mathematical theories can be freely used without introducing additional inconsistent information into inconsistency robust empirical theories that will be the core of future Intelligent Systems.

Direct Logic [<https://hal.archives-ouvertes.fr/hal-01566393>] derives its name because it directly categorically axiomatizes the Natural Numbers, Real Numbers, Ordinal Numbers, Set Theory, the Lambda Calculus, and Actors in sense that up to a unique isomorphism, there is only one model that satisfies axioms of the respective theories. Direct Logic is also called “direct” because it directly deals with propositions instead of attempting to deal with them indirectly using Gödel numbers. Propositions in Direct Logic must be uncountable (explained below) in order to provide categorical axiomatizations of the Natural Numbers, etc. Consequently, it is impossible to give a Gödel number to every proposition.

In Direct Logic, categorical theories of the Natural Numbers, Real Numbers, Ordinal Numbers, Set Theory, the Lambda Calculus, and Actors are inferentially decidable, meaning that every true proposition is provable and every proposition is either provable or disprovable. Also each of these theories self proves that it is *formally* consistent in the sense it proves that there is no proposition that is both provable and disprovable. (Of course, this does not *per se* prove *constructive* consistency meaning that no contradiction can be derived from stated axioms and rules of inference in Direct Logic.) Proving formal consistency of these theories does not contradict Gödel’s results because strong types mean that Gödel’s famous proposition “*I’m unprovable.*” cannot be constructed using the Diagonal Lemma fixed point because of strong parameterized types, e.g., *Proposition* $\langle 2 \rangle$ where the type *Proposition* has parameter 2 meaning a 2nd order proposition in the sense of higher order logic and \langle (read as “begin parameters”), \rangle (read as “end parameters”) are used to delimit the parameters of a type. In Direct Logic, Gödel 1931 results can be formalized as follows using the following definition of NotProvable: NotProvable $[\Psi] \equiv \neg \vdash \Psi$. The construction of *I’mUnprovable* is blocked because NotProvable does not have a fixed point *I’mUnprovable* such that *I’mUnprovable* $\Leftrightarrow \neg \vdash I’mUnprovable$ because NotProvable maps a proposition Ψ of order n into the proposition $\neg \vdash \Psi$ of order $n+1$. The Liar’s Paradox *I’mFalse* $\Leftrightarrow \neg I’mFalse$ is blocked in the same way.

Alonzo Church became alarmed about the new science of computation that he was developing with colleagues and so published the following concern: “Indeed, if there is no formalization of logic as a whole, then there is no exact description of what logic is, for it in the very nature of an exact description that it implies a formalization. And if there no exact description of logic, then there is no sound basis for supposing that there is such a thing as logic.” [The Richard Paradox. 1934] By “formalization of logic as a whole”, Church meant that theorems must be computationally enumerable. However, the following categorical induction axiom for the theory **Nat** of the Natural Numbers \mathbb{N} has uncountably many instances:

$\forall [P: \textit{Proposition} \langle \mathbb{N} \rangle] (P[0] \wedge \forall [i: \mathbb{N}] P[i] \Rightarrow P[+_1[i]]) \Leftrightarrow \forall [i: \mathbb{N}] P[i]$ where $+_1$ is the successor function. Although theorems of **Nat** cannot be computationally enumerated, the theory is “effective” in the sense that proof checking is computationally decidable.

For example in the theory **Nat**, there are uncountable proof checkers of the form $\text{ForAllEliminationChecker}[c]$ where $t: \text{Type} \langle \text{Nat} \rangle$ and $c:t$ such that

$\text{ForAllEliminationChecker}[c].[\Psi_1, \Psi_2] \equiv \Psi_1$ if $(\forall [x:t] P[x])$ then $\Psi_2 = P[c]$ else **False**
 // If Ψ_1 is $\forall [x:t] P[x]$, then $\Psi_2 = P[c]$, otherwise **False**

Consequently, $(\forall [x:t] P[x]) \vdash \frac{\text{ForAllEliminationChecker}[c]}{\text{Nat}} P[c]$

1st order logic is not suitable for pervasively inconsistent information in Intelligent Systems because presence of inconsistency can infer invalid results. Also, concurrent systems can implement unbounded nondeterminism. However, unbounded nondeterminism violates compactness making 1st order logic unsuitable as a tool. Furthermore, 1st order logic is unsuitable even for basic mathematical theories such as the Natural Numbers. The following is true of the Natural Numbers:

Instance Adequacy: $(\forall [i:\mathbf{N}] \vdash P[i]) \Rightarrow \forall [i:\mathbf{N}] P[i]$

However, a 1st order theory of the Natural Numbers **Nat₁** is inconsistent if the above true proposition is added as schema to the usual Dedekind/Peano axioms along with a first order axiomatization of \vdash . In developing a mathematical theory of computation, Church and his colleagues were very much concerned about Cantor's Diagonal Argument. However, provably total procedures could be subject to Cantor's argument because if theorems are computationally enumerable by a provably total procedure, then the following contradiction arises: provably computable real numbers between 0 and 1 represented as **Boolean^N** are computationally enumerable by a **Nat₁** provably total procedure $\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}$ because

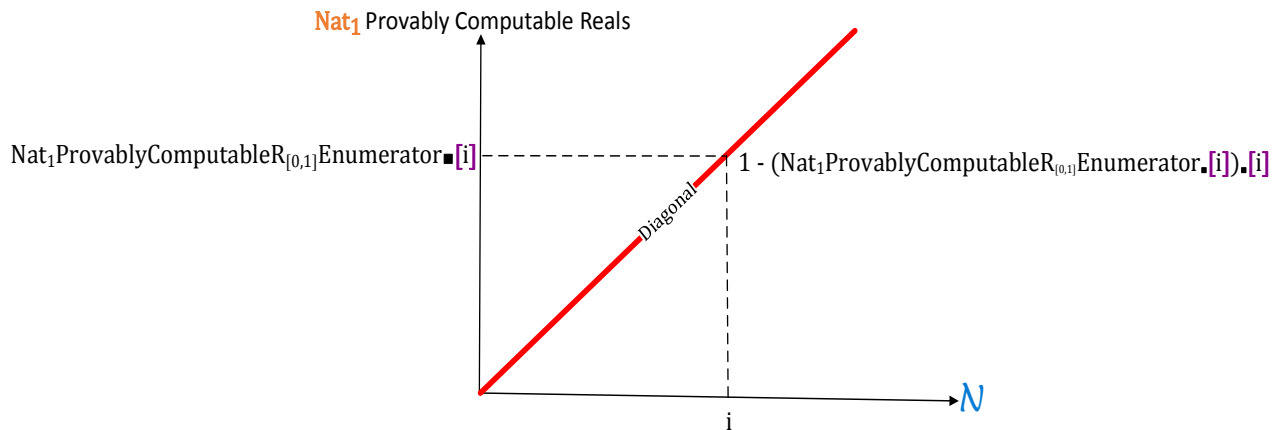
$\forall [i:\mathbf{N}] \vdash \text{Total}[\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}.[i]]$ implies

$\vdash_{\text{Nat}_1} \forall [i:\mathbf{N}] \text{Total}[\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}.[i]]$ using the above Instance Adequacy schema. Therefore the following is a **Nat₁** provably total procedure:

$\text{Diagonal}.[i:\mathbf{N}]:\text{Boolean} \equiv 1 - (\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}.[i]).[i]$

Thus Diagonal is a **Nat₁** provably computable real number between 0 and 1 that is not enumerated by $\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}$, which is a contradiction.

Cantors Diagonal Argument is diagrammed below:



$\text{Diagonal}.[i:\mathbf{N}]:\text{Boolean} \equiv 1 - (\text{Nat}_1\text{ProvablyComputableR}_{[0,1]}\text{Enumerator}.[i]).[i]$

Choice-free Stone duality

Wesley H. Holliday

University of California, Berkeley

The standard topological representation of a Boolean algebra via the clopen sets of a Stone space requires a nonconstructive choice principle, equivalent to the Boolean Prime Ideal Theorem. In this talk, reporting on joint work with Nick Bezhanishvili, I will describe a choice-free topological representation of Boolean algebras. This representation uses a subclass of the spectral spaces that Stone used in his representation of distributive lattices via compact open sets. We prove without choice principles that any Boolean algebra arises from a special spectral space X via the sets that are both compact open in X and regular open in the upset topology of the specialization order of X , taking advantage of Tarski's observation that the regular opens of any topological space form a Boolean algebra. Our representation is therefore a mix of Stone and Tarski, with the two connected by Vietoris: the relevant spectral spaces also arise as the hyperspace of nonempty closed sets of a Stone space endowed with the upper Vietoris topology. This connection makes clear the relation between our point-set topological approach to choice-free Stone duality, which may be called the hyperspace approach, and a point-free approach to choice-free Stone duality using Stone locales. Unlike Stone's representation of Boolean algebras via Stone spaces, our choice-free topological representation of Boolean algebras does not show that every Boolean algebra can be represented as a field of sets; but like Stone's representation, it provides the benefit of a topological perspective on Boolean algebras, only now without choice. In addition to representation, we establish a choice-free dual equivalence between the category of Boolean algebras with Boolean homomorphisms and a subcategory of the category of spectral spaces with spectral maps. We show how this duality can be applied by using it to prove some basic facts about Boolean algebras.

COMPUTATIONAL COMPLEXITY OF CHECKING VARIOUS SEMIGROUP PROPERTIES

July 14, 2018

TREVOR JACK

ABSTRACT. We introduce a method of investigating semigroup properties using graph theory. Given a transformation semigroup generated by transformations a_1, \dots, a_k on n letters, we produce a graph with n^d vertices and directed edges corresponding to the generators acting on the vertices component-wise. The number of components, d , depends upon which semigroup property we are investigating. We will sketch several results obtained thus far, including: (a) nilpotency, (b) regularity, and (c) identity satisfaction.

The sum of observables on a σ -distributive lattice effect algebra

Jiří Janda¹ and Yongming Li²

^{1,2} College of Mathematics and Information Science,
Shaanxi Normal University, Xi'an 710062, People's Republic of China
E-mail: jjanda@mail.muni.cz yjxie@snnu.edu.cn ¹

Abstract

In our work, we focus on a recently presented structure of a lattice ordered semigroup $(\mathcal{BO}(E); +, \leq)$ on the set of bounded observables $\mathcal{BO}(E)$ on a σ -distributive lattice effect algebra E . On this structure, the lattice order \leq is given by Olson order [1] and the semigroup operation $+$, so called *sum* of observables [2], is a natural generalization of the usual point-wise sum of random variables in the classical case.

We describe a spectrum $\sigma(x + y)$ of the sum $x + y$ of bounded observables $x, y \in \mathcal{BO}(E)$ and we show that $x + y$ can be expressed by using spectral resolutions of x and y in points of a spectrum $\sigma(x)$ of one of its summands. We provide conditions under which the operation $+$ preserves continuity of spectral resolutions of observables x and y . Moreover, we introduce a notion of a *dense* observable as a dual notion to a meager observable [5] and we describe relations between sets of dense and meager observables.

References

- [1] Dvurečenskij, A.: *Olson Order of Quantum Observables*, Int. J. Theor. Phys. **55**, Issue 11, 4896–4912 (2016).
- [2] Dvurečenskij, A.: *Sum of Observables on MV-effect Algebras*, Soft Comput. (2017), <https://doi.org/10.1007/s00500-017-2741-1>.

¹Keywords: Effect algebra, quantum observable, sum of observables, spectral resolution, Olson order, meager observable, dense observable.

- [3] Dvurečenskij, A., Kuková, M: *Observables on Quantum Structures*, Inf. Sci. **262**, 215–222 (2014).
- [4] Foulis, D. J., Bennett, M. K., *Effect Algebras and Unsharp Quantum Logics*, Found. Phys. **24**, 1331–1352 (1994).
- [5] Janda, J., Xie, Y.: *The spectrum of the sum of observables on σ -complete MV-effect algebras*, Soft Comput. (2018), <https://doi.org/10.1007/s00500-018-3078-0>.

- SÁNDOR JENEI, *Group representation, Hahn-type embedding, and the FSSC for \mathbf{IUL}^{fp}* . Institute of Mathematics and Informatics, University of Pécs, Ifjúság u. 6., Pécs, Hungary.

E-mail: jenei@ttk.pte.hu.

URL Address: <http://jenei.ttk.pte.hu/home.html>.

An FL_e -algebra [2] is a structure $\mathbf{X} = (X, \wedge, \vee, \otimes, \rightarrow, t, f)$ such that (X, \wedge, \vee) is a lattice, (X, \leq, \otimes, t) is a commutative monoid, and f is an arbitrary constant. One defines $x' = x \rightarrow \otimes f$ and calls \mathbf{X} involutive if $(x')' = x$ holds. Call \mathbf{X} group-like if it is involutive and $t = f$. Since for involutive FL_e -chains $t' = f$ holds, one extremal situation is the integral case (when t is the top element of the universe and hence f is its bottom one) and the other extremal situation is the group-like case (when the two constants coincide). Prominent classes of group-like FL_e -chains are totally-ordered Abelian groups and odd Sugihara chains, the latter constitute an algebraic semantics of a logic at the intersection of relevance logic and fuzzy logic. These two classes are also extremal in the sense that lattice-ordered Abelian groups have a single idempotent element, whereas all elements of any odd Sugihara algebra are idempotent.

Hahn's celebrated embedding theorem asserts that linearly ordered Abelian groups embed in the lexicographic product of real groups. Conrad, Harvey, and Holland generalized Hahn's theorem for lattice-ordered Abelian groups [1]. We prove a representation theorem for group-like FL_e -chains which possess only finitely many idempotents. The representation uses only linearly ordered Abelian groups and a newly introduced construction, called partial lexicographic product. As a corollary we extend Hahn's theorem to this class of semigroups by showing that any such algebra embeds in some finite partial-lexicographic product of linearly ordered Abelian groups. As an application for this embedding, we show the finite strong standard completeness of an axiomatic extension of the Involutive Uninorm Logic \mathbf{IUL} [4] by $\mathbf{t} \Leftrightarrow \mathbf{f}$.

[1] P. F. CONRAD, J. HARVEY, W. C. HOLLAND, *The Hahn embedding theorem for lattice-ordered groups*, *Transactions of the American Mathematical Society*, vol. 108 (1963), pp. 143–169.

[2] N. GALATOS, P. JIPSEN, T. KOWALSKI, H. ONO, *Residuated Lattices: An Algebraic Glimpse at Substructural Logics*, Studies in Logic and the Foundations of Mathematics, Elsevier, 2007.

[3] H. HAHN, *Über die nichtarchimedischen Grössensysteme*, *Sitzungsberichte der Akademie der Wissenschaften, Wien, Mathematik-Naturwissenschaften Klasse*, vol. 116 (1907), no. IIa, pp. 601–655.

[4] G. METCALFE, F. MONTAGNA, *Substructural fuzzy logics*, *Journal of Symbolic Logic*, vol. 72 (2007), no. 3, pp. 834–864.

CONVEX EFFECT ALGEBRAS AND THE KALMBACH MONAD

GEJZA JENČA

In [6] Kalmbach proved that for every bounded lattice L there is an orthomodular lattice $K(L)$ and a bounded lattice embedding $\eta_L: L \rightarrow K(L)$. Later, Mayet and Navara proved [7] that Kalmbach's result can be extended to bounded posets: for every bounded poset P there is an orthomodular poset $K(P)$ and a bounded poset embedding $\eta_P: P \rightarrow K(P)$. In [4], Harding proved that K is a functor from the category of bounded posets \mathbf{BPos} to the category of orthomodular posets \mathbf{OMP} and that K is left adjoint to the forgetful functor $U: \mathbf{BPos} \rightarrow \mathbf{OMP}$. Since U and K form an adjoint pair of functors, they induce a monad $T = U \circ K$ on \mathbf{BPos} . This monad is called the *Kalmbach monad*. In [5], Jenča proved that the category of effect algebras [2] is equivalent to the Eilenberg-Moore category of the Kalmbach monad.

We show that there is another monad S on \mathbf{BPos} , that can be described as a monad arising from the free multiplicative action of the real unit interval on bounded posets. The pair T, S satisfies the distributive laws in the sense of [1]. Therefore, $T \circ S$ is an underlying endofunctor of a monad. We prove that this monad arises from the free-forgetful adjunction between bounded posets and convex effect algebras [3]. However, the Eilenberg-Moore category of the monad $T \circ S$ is not equivalent to the category of convex effect algebras.

REFERENCES

- [1] Jon Beck. Distributive laws. In *Seminar on triples and categorical homology theory*, pages 119–140. Springer, 1969.
- [2] D.J. Foulis and M.K. Bennett. Effect algebras and unsharp quantum logics. *Found. Phys.*, 24:1325–1346, 1994.
- [3] S Gudder, S Pulmannová, S Bugajski, and E Beltrametti. Convex and linear effect algebras. *Reports on Mathematical Physics*, 44(3):359–379, 1999.
- [4] John Harding. Remarks on concrete orthomodular lattices. *International Journal of Theoretical Physics*, 43(10):2149–2168, 2004.
- [5] Gejza Jenča. Effect algebras are the Eilenberg-Moore category for the Kalmbach monad. *Order*, 32(3):439–448, 2015.
- [6] Gudrun Kalmbach. Orthomodular lattices do not satisfy any special lattice equation. *Archiv der Mathematik*, 28(1):7–8, 1977.
- [7] René Mayet and Mirko Navara. Classes of logics representable as kernels of measures. *Contributions to General Algebra*, 9:241–248, 1995.

NONASSOCIATIVE RIGHT HOOPS

PETER JIPSEN AND MICHAEL KINYON[†]

Abstract. A *right-residuated magma* is a partially ordered algebra $(A, \leq, \cdot, /)$ such that (A, \leq) is a poset, $\cdot, /$ are binary operations and $xy \leq z \iff x \leq z/y$. We define *nonassociative right hoops*, or *narhoop* for short, as right-residuated magmas such that

$$x \leq y \iff (x/y)y = x = (y/x)x.$$

Well known subclasses of narhoops are *right quasigroups* where \leq is the equality relation, and *right hoops*, defined by the identity $x/(yz) = (x/z)/y$ and the quasi-inequation $(x/y)y = x \Rightarrow x \leq y$.

Denoting the term $(x/y)y$ by $x \wedge y$, we prove that narhoops form a finitely based variety, axiomatized by the following identities:

$$\begin{aligned} (x \wedge y) \wedge x &= x \wedge y & xz \wedge (x \wedge y)z &= (x \wedge y)z \\ xy/y \wedge x &= x & (x/z) \wedge (x \wedge y)/z &= (x \wedge y)/z. \end{aligned}$$

This equational basis is independent as can be seen from algebras $A_i = \{0, 1\}$ ($i = 1, 2, 3, 4$) that each satisfy all but one of the above identities.

For A_1 , \cdot is ordinary multiplication and $x/y = y$.

For A_2 , $x \cdot y = x$ and $x/y = 1$.

For A_3 , $x \cdot y$ is addition modulo 2 and $x/y = 0$ except that $1/0 = 1$.

For A_4 , $x \cdot y$ is the max operation and x/y is addition modulo 2.

In general, neither \cdot nor the term operation \wedge of a narhoop is associative. However \wedge is associative in both right quasigroups and right hoops. In right quasigroups this follows from the identity $x \wedge y = x$. In right hoops \wedge turns out to be a semilattice operation ([1], Lem. 4).

We characterize the subvarieties in which the operation $x \wedge y = (x/y)y$ is associative and/or commutative. Narhoops with a left unit are shown to be integral if and only if \wedge is commutative, and their congruences are determined by the equivalence class of the left unit.

This research was supported by the automated theorem prover PROVER9 and the finite model builder MACE4, both created by McCune [2]. We would like to thank Bob Veroff for hosting the 2016 Workshop on Automated Deduction and its Applications to Mathematics (ADAM) which is where our collaboration began.

REFERENCES

- [1] P. Jipsen, On generalized hoops, homomorphic images of residuated lattices, and (G)BL-algebras, *Soft Computing* **21**(1) (2017), 17–27.
- [2] W. McCune, *Prover9 and Mace4*, version 2009-11A. <http://www.cs.unm.edu/~mccune/prover9>.

(Jipsen) FACULTY OF MATHEMATICS, SCHOOL OF COMPUTATIONAL SCIENCES, CHAPMAN UNIVERSITY, 545 W. PALM AVE, ORANGE, CA 92866 USA
E-mail address: jipsen@chapman.edu

(Kinyon) DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DENVER, DENVER, CO 80208 USA
E-mail address: mkinyon@du.edu

[†] Partially supported by Simons Foundation Collaboration Grant 359872.

A constructive view of weak topologies on a topos

Zeinab Khanjanzadeh

Ali Madanshekaf

Department of Mathematics

Faculty of Mathematics, Statistics and Computer Science

Semnan University

Semnan

Iran

emails: z.khanjanzadeh@gmail.com

amadanshekaf@semnan.ac.ir

Abstract

It is well known that the set of all Lawvere-Tierney topologies (or topologies, for short) on an elementary topos \mathcal{E} is not closed under the formation of composition. In fact, the composition two of them preserves the top element of the subobject classifier Ω of \mathcal{E} and preserves binary meets too, but lacks to be idempotent. From this point of view, it is natural to study topologies without idempotency on an arbitrary topos. This notion is called *weak topology* by Hosseini and et al. in [3].

In this talk we will study some properties related to this notion on the topos \mathcal{E} . We show that the set of all weak topologies on \mathcal{E} constitute a complete resituated lattice provided that \mathcal{E} is (co)complete. Also, when the weak Lawvere-Tierney topology on \mathcal{E} preserves binary meets we give an explicit description of the (restricted) associated sheaf functor on \mathcal{E} .

References

- [1] F. Borceux, Handbook of Categorical Algebra, Vol. I and III, Cambridge University Press, (1994).
- [2] D. Dikranjan and W. Tholen, Categorical Structure of Closure Operators, Kluwer, Netherlands, (1995).
- [3] S. N. Hosseini and S. SH. Mousavi, A Relation Between Closure Operators on a Small Category and Its Category of Presheaves, Appl. Cat. Struct., 14 (2006), 99-110.
- [4] P. T. Johnstone, Sketches of an Elephant: a Topos Theory Compendium, Vol. 1, Clarendon Press, Oxford, (2002).
- [5] S. Mac Lane and I. Moerdijk, Sheaves in Geometry and Logic, Springer-Verlag, New York, (1992).

THE LOGIC OF Σ FORMULAS

ANDRE KORNEILL

Bounded quantifiers are a standard feature of both finitary and transfinitary foundational systems. Each bounded quantifier may be interpreted as ranging over a completed portion of the universe, which is itself conceived to be an incomplete totality. In this conceptual framework, the true Σ formulas stand out as the formulas that become true in a completed portion of the universe.

We interpret each theory T , consisting of implications between Σ formulas, to be a deductive rewriting system for Σ formulas. Adding logical axioms to T , we obtain conservation theorems for classical, intuitionistic, and minimal logics. In this context, classical reasoning is characterized by a distributive law, suggesting a novel account of intuitionistic logic.

Natural theories of arithmetic and of set theory can be formalized in this deductive system. In both cases, the theory may be chosen to include a satisfaction predicate for all Σ formulas – all the formulas that may appear in a deduction.

Positive Group Homomorphisms of Free Unital Abelian ℓ -groups

Tomáš Kroupa^{1*}

The Czech Academy of Sciences, Institute of Information Theory and Automation
kroupa@utia.cas.cz

Normalized positive group homomorphisms $s: G \rightarrow \mathbb{R}$ (a.k.a. states) of a unital partially ordered Abelian group G form a natural dual object for G , the so-called state space of G . An important line of development for ordered groups has related interpolation groups to the partially ordered groups of all affine continuous real-valued functions on their state spaces. In particular, it is known that a unital partially ordered Abelian group is Archimedean if, and only if, its affine representation map is a unital positive isomorphism.

Baker-Beynon duality provides a representation of any free unital Abelian ℓ -group G by piecewise linear functions with integer coefficients. States of G are then in one-to-one correspondence with regular Borel probability measures over $\text{MaxSpec } G$. When the co-domain \mathbb{R} of a state is replaced with a Dedekind complete ℓ -group H , the resulting group of all (relatively bounded) positive group homomorphisms is lattice ordered and also Dedekind complete. On the other hand, relatively little is known about the structure of unital positive group homomorphisms $G \rightarrow H$, where H is not necessarily Dedekind complete. In this contribution we will make first steps towards a description of unital positive group homomorphisms $G \rightarrow G$ of any free unital Abelian ℓ -group G by (dual) continuous maps from $\text{MaxSpec } G$ into the state space of G . Our main tools involve characterization of discrete states and arithmetics of piecewise linear functions with integer coefficients.

*The author was supported by the GAČR project n. 17-04630S.

Some Aspects of Weak Ideal Topology on Acts over Monoids

Ali Madanshekaf

Department of Mathematics

Faculty of Mathematics, Statistics and Computer Science

Semnan University

Semnan

Iran

email: amadanshekaf@semnan.ac.ir

Abstract

A representation of a semigroup S by transformations of a set defines an S -act just as a representation of a ring R by endomorphisms of an abelian group defines an R -module. More precisely, a right act A_S over the monoid S is a set A for which a “scaller product” $as \in A$ for $s \in S$ and $a \in A$ is defined such that for all $s_1, s_2 \in S$, $a \in A$ one has $a(s_1s_2) = (as_1)s_2$ and $a1 = a$ if S has an identity 1. Left S -acts are defined analogously.

In this talk we introduce and study the weak ideal topology j^I on the category of right acts of a given monoid S , for a left (and sometimes central) ideal I of S . Furthermore, some relevant examples and applications are presented.

References

- [1] K. Ahmadi, A. Madanshekaf, Nakayamas lemma for acts over monoids, *Semigroup Forum* (2015) 91:321337.
- [2] D. Dikranjan and W. Tholen, *Categorical Structure of Closure Operators*, Kluwer, Netherlands, (1995).
- [3] S. N. Hosseini and S. SH. Mousavi, A Relation Between Closure Operators on a Small Category and Its Category of Presheaves, *Appl. Categ. Struc.*, 14 (2006), 99-110.
- [4] M. Kilp, U. Knauer and A.V. Mikhalev, *Monoids, Acts and Categories: With Applications to Wreath Products and Graphs*, Walter de Gruyter, Berlin, (2000).
- [5] S. Mac Lane and I. Moerdijk, *Sheaves in Geometry and Logic*, Springer-Verlag, New York, (1992).

On the universal–algebraic theory of measure and integration

Vincenzo Marra
Dipartimento di Matematica “Federigo Enriques”
Università degli Studi di Milano
Milan, Italy

The algebra of measure and integration has been investigated in the 20th century by several authors, including Daniell, Carathéodory, and Segal. In this talk I would like to survey some results obtained in the recent programme of developing the universal (i.e., equational) algebra of integration theory. The programme splits neatly into a finitely additive and a countably additive version. In the latter case one needs to use algebras with operations of countably infinite arity, so that in particular no Subdirect Representation Theorem is available. In the former, finitary universal algebra suffices. In both cases the theory is far more natural if one allows two–sorted algebras: the first sort for measurable functions, and the second sort for the values of their integral with respect to some measure. The results available so far in the countably additive case are due to Marco Abbadini's work towards his Ph.D. thesis, and are presented in his contributed talk. Here, on the other hand, I summarise some of the results obtained for finitely additive expectations of random variables with respect to probability measures. One main fact in this context is that the ensuing variety of two–sorted algebras is generated by a single algebra consisting of the rational–valued functions on the Cantor space, where the rationals are equipped with the discrete topology, and the Cantor space is equipped with an appropriate "generic" Borel probability measure. This result, for which we know no satisfactory analogue in the countably additive case, was obtained jointly with Tomas Kroupa. If time allows I will sketch the next steps that we would like to take in this research programme.

Homeomorphisms of Čech-Stone remainders: the zero-dimensional case

Paul McKenney

August 1, 2018

Let X be a locally compact, noncompact topological space, and let $X^* = \beta X \setminus X$ be its Čech-Stone remainder. To what extent does X^* “remember” X ? That is, if $X^* \simeq Y^*$, how different can X and Y be? I will show that, assuming some combinatorial consequences of the proper forcing axiom, the answer is “not much”; specifically, if X and Y are zero-dimensional, locally compact Polish spaces, then $X^* \simeq Y^*$ if and only if X and Y have homeomorphic cocompact subspaces, and moreover, a homeomorphism between X^* and Y^* must be induced by a homeomorphism between such subspaces of X and Y . Time-permitting, I will discuss open problems relating to other spaces and C^* -algebras. This represents joint work with Ilijas Farah.

Uniform Interpolation

George Metcalfe

Mathematical Institute
University of Bern, Switzerland
`george.metcalfe@math.unibe.ch`

Uniform interpolation, a sharpened version of the well-known Craig interpolation property, is a property of consequence in logical systems possessing intriguing connections to concepts from universal algebra, proof theory, and model theory. In this tutorial, we will discover and explore these connections in some detail. As a motivating case study, we will consider Pitts' (proof-theoretic) proof of uniform interpolation for intuitionistic propositional logic and subsequent applications to the model theory of Heyting algebras by Ghilardi and Zawadowski. We then proceed to a more general universal-algebraic setting, relating uniform interpolation for a class of algebras to amalgamation, coherence, and fixpoint properties, and the existence of model completions. Key examples of classes of algebras that do or do not admit these properties will include various varieties of lattices, Boolean algebras with operators, and residuated lattices.

BETWEEN TUKEY EQUIVALENCE AND BOOLEAN AUTOMORPHISM

DAVID MILOVICH

ABSTRACT. Two directed posets D, E are Tukey equivalent if and only if D and E both order-embed as cofinal subsets of a poset P . For D and E directed subsets of a Boolean algebra A , I will introduce pin equivalence, which requires exactly that, in some Boolean algebra A' extending A , there is a (Boolean) automorphism h of A' that maps the ideal generated by D to the ideal generated by E . Even though A' may depend on D and E , pin equivalence is transitive. And it strictly implies Tukey equivalence.

A longstanding open problem about homogeneous compact Hausdorff spaces has a Boolean analog: does every Boolean algebra B extend to a Boolean algebra C such that, for every pair of prime ideals P, Q of C , there is an automorphism of C sending P to Q ? (This is open in all models of ZFC.) Pin equivalence is a promising concept for proving partial results related to this question because we can flexibly strengthen parts of its definition. For example, we could require A to be relatively complete in A' , or we could require h to also map the Boolean closure of D onto the Boolean closure of E . As a beginning of this research program, I will state a few results about how pin equivalence interacts with coproducts and with the countable separation property.

E-mail address: ultrafilter@gmail.com

URL: <http://dkmj.org>

WELKIN SCIENCES, 2 N NEVADA AVE, COLORADO SPRINGS, CO 80903, USA

Key words and phrases. Tukey equivalence, pin equivalence, compact homogeneous space.

SUPERNILPOTENCE IN TAYLOR VARIETIES

ANDREW MOORHEAD

ABSTRACT. Both the classical term condition that defines the binary commutator and its generalization to a condition that defines the higher commutator are nicely described as a condition on a certain algebra of n -dimensional cubes. If the variety under consideration is permutable, these cubes have an interesting property: interpreting each of them as a pair of lower dimensional cubes produces a congruence and this is true for each way of decomposing an n -dimensional cube into a pair of hyperfaces. Similar relations describe the commutator for modular varieties. We will define what we call an n -dimensional congruence, discuss how properties of ordinary congruences generalize to higher dimensions and demonstrate that these higher dimensional congruences provide a useful framework to study commutators by showing that a supernilpotent algebra that belongs to a Taylor variety is nilpotent.

The Role of Bitopology in Natural Duality

M. Andrew Moshier

July 13, 2018

BITOPOLOGICAL REASONING features in

- The reals, where the standard Euclidean topology does not yield ordering, but the upper and lower topologies do.
- The upper and lower Vietoris topologies on hyperspaces $H \subseteq \mathcal{P}(X)$. One has basis given by $\square U = \{K \in \mathcal{K}(X) \mid K \subseteq U\}$, the other by $\diamond U = \{K \in \mathcal{K}(X) \mid U \cap K \neq \emptyset\}$ for open U .
- The reals with Euclidean topology and de Groot topology, consisting of complements of compact subsets (together with \emptyset).
- In domain theory, the Scott topology with its Lawson dual.
- Natural dualities for partial order-based algebras, such as Priestley duality (for bounded distributive lattices), Banaschewski duality (for posets) and Mislove-Stralka duality (for semi-lattices).

Based on the theory of frames we introduce a natural duality for bitopological spaces. The central concept is that of a *d-frame*, which axiomatises the two interaction of two open set lattices on a set.

We give accounts of natural notions of regularity, normality and compactness for bitopological spaces, and their manifestations in *d-frames*. This yields the machinery to locate precisely within this general landscape a number of classical Stone-type dualities. We also prove a Čech-Stone result characterizing all bitopological compactifications. A surprising and somehow satisfying result of this is that the real numbers possess exactly one bitopological compactification, which supplies two points. In other words, when we take order into account, the reals can only be compactified in one way by adding $-\infty$ and ∞ .

The general duality of *d-frames* and bitopological spaces can be given a logical reading by viewing the open sets of one topology as *positive* extents of formulas, and those of the other topology as *negative* extents. This point of view emphasises the fact that formulas may be undecidable in certain states and may be self-contradictory in others. We also obtain two natural orders on the set of formulas, one related to Scott's information order and the other being the usual logical implication. The interplay between the two is one of the main organising principles of the lectures.

We end by returning to classical Priestley and Stone duality, giving properly bitopological accounts of both, and extending them to all compact Hausdorff spaces (ordered spaces in the case of Priestley).

Parts of biframes and d-frames

M. Andrew Moshier, Imanol Mozo Carollo
and Joanne Walters-Wayland

May 15, 2018

The most studied pointfree approach to bitopological spaces was presented by Banaschewski, Brummer and Hardie in [1]. There pointfree bitopological spaces are *biframes*. The well-known dual adjunction between spaces and frames naturally extends to one between the category \mathbf{BiSp} of bispaces and the category \mathbf{BiFrm} of biframes. The adjunction, however, no longer is a Stone-type duality with a dualising object carrying the structure of both categories. This issue was addressed by Jung and Moshier in [2] by the category \mathbf{dFrm} of d-frames. The objects $(L_-, L_+, \mathbf{con}, \mathbf{tot})$ consist of two frames L_- and L_+ together with two relations \mathbf{con} and \mathbf{tot} thought of as abstractions of disjointness and covering relations of open sets of the space. The two relations are governed by axioms that capture this intuition. Morphisms are the obvious ones: pairs of frame morphisms that preserve disjointness and covering.

The distinctions between d-frames and biframes is especially highlighted by their differences with respect to subspaces. Specifically, extremal epimorphisms in the two categories are quite different. In this talk we will explore these differences with an eye on a further investigation of the notion of *pointfree bitopological spaces*.

REFERENCES

- [1] B. Banaschewski, G. C. L. Brummer, K. A. Hardie, Biframes and bispaces, *Quaest. Math.* **6**, (1983) 13–25.
- [2] A. Jung, M. A. Moshier, *On the bitopological nature of Stone duality* Technical Report CSR-06-13, School of Computer Science, The University of Birmingham (2006).

Ramsey spaces and the Katetov order

Sonia Navarro Flores, UNAM

Abstract. Some Borel ideals \mathcal{I} , such as \mathcal{ED}_{fin} and $Fin \times Fin$ satisfy that \mathcal{I}^+ , the collection of all the \mathcal{I} -positive sets, contains a Ramsey space in a dense way. If \mathcal{I}, \mathcal{J} are a ideals over countable sets, it is difficult to prove that \mathcal{I} and \mathcal{J} are not Katetov equivalent ideals. In this talk we will see how the Ramsey structure of Ramsey spaces contained in \mathcal{I}^+ and \mathcal{J}^+ is helpful to prove that \mathcal{I} and \mathcal{J} are nor Katetov equivalent ideals.

On injective constructions of S -semigroups

PASEKA Jan¹ and ZHANG Xia²

¹ *Department of Mathematics and Statistics, Faculty of Science, Masaryk University
Kotlářská 2, 611 37 Brno
Czech Republic
E-mail: paseka@math.muni.cz*

² *School of Mathematical Sciences, South China Normal University
510631 Guangzhou
China
E-mail: xzhang@m.scnu.edu.cn*

There are quite a lot of papers investigating injective hulls for algebras. Here we only mention some of them which our research is related to. Injective hulls for posets were studied by Banaschewski and Bruns ([1], 1967) where they got that the injective hull of a poset is its MacNeille completion. Later on, Bruns and Lakser constructed injective hulls of semilattices ([2], 1970), and the results were soon applied into S -systems over a semilattice by Johnson, Jr., and McMorris ([3], 1972). In 2012, Lambek, Barr, Kennison, Raphael ([4]) studied a kind of category of pomonoids in which normal category of pomonoids is its subcategory, and found injective hulls for pomonoids. Later on, Zhang and Laan generalized the results of Lambek, Barr, Kennison, Raphael into posemigroup case ([5], 2014), and then S -posets ([6], 2015) and ordered Ω -algebras ([7], 2016).

This paper devotes to investigations on injective constructions of S -semigroups. We will study injectivity with respect to a specific class of order-embeddings. We show that an S -semigroup is injective iff it is an S -semigroup quantale.

Our approach in this paper sheds a new light on applications of a new kind of quantale-like structure and establishes the injective hull for every S -semigroup.

References

- [1] B. Banaschewski, G. Bruns: *Categorical construction of the MacNeille completion*. Arch. Math. **XVIII** (1967), 369-377.
- [2] G. Bruns, H. Lakser: *Injective hulls of semilattices*. Canad. Math. Bull. **13** (1970), 115–118.
- [3] C.S. Johnson, Jr., F.R. McMorris: *Injective hulls on certain S -systems over a semilattice*. Proc. Amer. Math. Soc. **32** 1972, 371-375.
- [4] J. Lambek, M. Barr, J. F.Kennison, R. Raphael: *Injective hulls of partially ordered monoids*. Theory Appl. Categ. **26** (2012), 338–348.
- [5] X. Zhang, V. Laan: *Injective hulls for posemigroups*. Proc. Est. Acad. Sci. **63** (2014), 372–378.
- [6] X. Zhang, V. Laan: *On injective hulls of S -posets*. Semigroup Forum. **91** (2015), 62–70.
- [7] X. Zhang, V. Laan: *On injective hulls for ordered algebras*. Algebra Universalis. **76** (2016), 339–349.

Jan Pavlík

Distributivity of a segmentation lattice

Given a T1-closure space, i.e. a topped intersection structure including all singletons, by a segmentation we mean a partition of the underlying set into the closed sets. The segmentations form a lattice into which the original lattice of closed sets can be mapped. In this talk we focus on finitary spaces, i.e., those spaces where the closure of a set can be obtained as union of closures of its finite subsets. We characterize finitary spaces for which the lattice of segmentations is distributive. Their description is provided in terms of acyclicity and weaker versions of distributivity.

TERM OPERATIONS IN $\mathcal{V}(\mathbf{N}_5)$

JOHN W. SNOW

In [2] the author proved that every finite lattice in the variety generated by \mathbf{M}_3 is isomorphic to the congruence lattice of a finite algebra. For any finite set A , there is a set of special operations on the equivalence relations of A so that a lattice \mathbf{L} of equivalence relations on A is the congruence lattice of an algebra on A if and only if \mathbf{L} is closed under these special operations. These equivalence relation operations can be based on primitive positive definitions as in [2] or graphical compositions as in [3]. The approach used in [2] was to show that each of these equivalence relation operations is actually a term operation of the lattice of equivalence relations on the three element set. In [1] Hegedűs and Pálffy improve on this \mathbf{M}_3 result and introduce the notion of a congruence lattice being power hereditary. Suppose that \mathbf{L} is the congruence lattice of a finite algebra with universe A . \mathbf{L} is power hereditary if every 0-1 sublattice of \mathbf{L}^n is the congruence lattice of an algebra with universe A^n . It turns out that \mathbf{L} is power hereditary if and only if each of the special equivalence relation operations is actually a term operation of \mathbf{L} . It seems reasonable, therefore, to address the question, given an operation p on a finite lattice \mathbf{L} , what conditions must p satisfy in order to be a term operation of \mathbf{L} ? In this paper, we prove that in the variety generated by \mathbf{N}_5 , the term operations on finite lattices are exactly the order preserving operations which are preserved by endomorphisms.

REFERENCES

- [1] Hegedűs, P., Pálffy, P.: Finite modular congruence lattices. *Algebra Universalis* **54**, 105–120 (2005)
- [2] Snow, J.: Every finite lattice in $\mathcal{V}(\mathbf{M}_3)$ is representable. *Algebra Universalis* **50**, 75–81 (2003)
- [3] Werner, H.: Which partition lattices are congruence lattices? *Colloq. Math. Soc. Janos Bolyai* **14**, 433–453 (1976). (Lattice Theory, Szeged, 1974; eds A.P. Huhn and E.T. Schmidt), North Holland

Promise Constraint Satisfaction Problems

Athena Sparks

University of Colorado Boulder

Abstract

Given two relational structures \mathbb{A} and \mathbb{B} and a homomorphism between them, a promise constraint satisfaction problem (PCSP) over \mathbb{A} and \mathbb{B} is the problem of deciding whether the structure \mathbb{A} satisfies a given primitive positive sentence or the structure \mathbb{B} does not satisfy the sentence. For example, the problem of deciding whether a graph is 3-colorable or not even 10-colorable can be formulated as PCSP. We use an algebraic approach to study the time complexity of such problems. Using methods similar to those found in discussions of constraint satisfaction problems (CSPs), we speculate that the polymorphisms from \mathbb{A} to \mathbb{B} are the key to classifying the complexity of PCSPs. We will give a summary of results for PCSPs that are analogous to those found for CSPs and discuss the significant differences between CSPs and PCSPs that remain the obstacle for classification.

Keywords— complexity of promise problems, polymorphisms, clonoids

Closure operators on the clone lattice defined by ultrapowers

AGNES SZENDREI
(University of Colorado Boulder)

If C is a clone on a set A , let $\Gamma_\kappa(C)$ be the clone of all operations $f: A^m \rightarrow A$ that are interpolable on $< \kappa$ -many elements of A^m by operations in C . Let $\Gamma_\kappa^*(C)$ be the clone of all operations $f: A^m \rightarrow A$ with the property that the natural extension to any ultrapower, $f_{\mathcal{U}}: (A/\mathcal{U})^m \rightarrow A/\mathcal{U}$, is interpolable on $< \kappa$ -many elements of $(A/\mathcal{U})^m$ by extensions of operations in C .

I will characterize membership in $\Gamma_\kappa^*(C)$, and use the characterization to derive Vaggione's infinitary version of the Baker–Pixley Theorem. These are joint results with Keith Kearnes.

- ▶ TIMOTHY TRUJILLO, *Hypnatural numbers in ultra-Ramsey theory*.
Sam Houston State University
Huntsville, Texas 77341.
E-mail: trujillo@shsu.edu.

We present an alternative formalism to deal with ultrafilters grounded on the use of the infinite hypnatural numbers of nonstandard analysis, which - in a precise sense - replace the use of ultrafilters. We then use the formalism to prove a new infinite-dimensional extension of Ramsey's theorem for ultrafilter trees. We conclude by extending the result to the abstract setting of topological Ramsey spaces.

BOOLEAN-VALUED POINT-FREE TOPOLOGY II

JOSEPH VAN NAME

We shall investigate how frames in a ground model become frames in forcing extensions and how forcing preserves properties of frames. Suppose that M is a frame and B is a complete Boolean algebra which is a subframe of M . Then M is a B -valued structure where Boolean-valued equality and inequality are defined by $b \leq \|x = y\|$ iff $b \wedge x = b \wedge y$ and $b \leq \|x \leq y\|$ iff $b \wedge x \leq b \wedge y$. Furthermore,

$$V^B \models \text{“}M \text{ is a frame.”}$$

If L is a frame and B is a complete Boolean algebra, then the frame coproduct $L \oplus B$ contains B as a subframe, so

$$V^B \models \text{“}L \oplus B \text{ is a frame.”}$$

The frame $L \oplus B \in V^B$ should be thought of as the B -valued version of the frame L . The B -valued structure $L \oplus B \in V^B$ inherits topological properties from the frame L . For example,

- (1) L is connected and locally connected if and only if

$$V^B \models \text{“}L \oplus B \text{ is connected and locally connected,”}$$

- (2) L is regular if and only if

$$V^B \models \text{“}L \oplus B \text{ is regular,”}$$

- (3) if L is completely regular, then

$$V^B \models \text{“}L \oplus B \text{ is completely regular,”}$$

and

- (4) if L is paracompact, then

$$V^B \models \text{“}L \oplus B \text{ is paracompact.”}$$

Forcing extensions add points to frames since for each regular frame L , there is a complete Boolean algebra B such that

$$V^B \models \text{“}L \oplus B \text{ is a Polish space”}.$$

E-mail address: `jvannname@mail.usf.edu`

Uncountable Rainbow Ramsey Theory Revisited

Jing Zhang
Carnegie Mellon University

We will talk about some recent results on Rainbow Ramsey partition relations, continuing the lines of research by Galvin, Todorcevic, Abraham, Cummings and Smyth. Among others, the topics will contain:

- The rainbow Ramsey theorem for 2-bounded colorings does not characterize weakly compact cardinals, thus answering a question by Abraham, Cummings and Smyth.
- Rainbow Ramsey theorems for the successor of singular cardinals of countable cofinality.
- Some results on coloring triples.

These results serve as evidence that rainbow Ramsey theory is a “strict weakening” of Ramsey theory.

Author Index

Abbadini, Marco	1
Ahmed, Shehzad	2
Balcerzak, Marek	11
Ball, Richard	3
Bartosova, Dana	4
Berg, Kevin	5
Brian, Will	6
Busaniche, Manuela	7
Colacito, Almudena	8
Crockett, Eran	9
Cruz, Maria	10
Das, Pratulananda	11
De Meo, William	12
Ding, Yifeng	13
Dobrinen, Natasha	14
Dzamonja, Mirna	15
Evans, Matt	16
Farah, Ilijas	41
Fernandez-Breton, David	17
Filipzak, Malgorzata	11
Fisher, John	18
Freese, Ralph	12
Fussner, Wesley	20, 21
Galatos, Nick	22
Gartside, Paul	23
Hachtman, Sherwood	24
Harding, John	25
Hathaway, Daniel	14
Heunen, Chris	25
Hewitt, Carl	26
Holliday, Wesley	28
Jack, Trevor	29
Janda, Jiri	30
Jenei, Sándor	32
Jenča, Gejza	33
Jipsen, Peter	20, 34

Kearnes, Keith	53
Khanjanzadehseresti, Zeinab	35
Kinyon, Michael	34
Kornell, Andre	37
Kroupa, Tomas	38
Li, Yongming	30
Lindenhovius, Bert	25
Madanshekaf, Ali	35, 39
Marra, Vincenzo	40
Mayr, Peter	29
Mckenney, Paul	41
Metcalfe, George	42
Milovich, David	43
Moorhead, Andrew	44
Moshier, M. Andrew	45, 47
Mozo Carollo, Imanol	47
Navara, Mirko	25
Navarro Flores, Sonia	48
Paseka, Jan	49
Pavlik, Jan	50
Prikry, Karel	14
Prenosil, Adam	22
Sinapova, Dima	24
Snow, John	51
Sparks, Athena	52
Swaczyna, Jaroslav	11
Szendrei, Agnes	53
Trujillo, Timothy	54
Ugolini, Sara	21
Valeriotte, Matthew	12
Van Name, Joseph	55
Walters-Wayland, Joanne	47
Zhang, Jing	56
Zhang, Xia	49