BLAST 2008

University of Denver
Denver, CO, USA

August 6–10, 2008

Contents

Workshop on forbidden configurations in Priestley spaces

Richard N. Ball
University of Denver
rball@du.edu
Coauthors: Ales Pultr (Charles University)

1. A brief review of Priestley duality, including the full-blown version.
2. Configurations. Forbidden configurations and resulting classes of distributive lattices. A few classical results and problem setting.
3. Generating first-order formulas by forbidden trees.
4. First negative result: the diamond.
5. Coproducts of Priestley spaces. Their existence and general form. The coproductivity problem.
6. Los' Theorem and closing the first cycle:
For a topped configuration P the following statements are equivalent:

- P is acyclic,
- P is coproductive,
- forbidding P in $\mathcal{P}(L)$ creates a first-order class of distributive lattices.

7. Aside: Prohibiting configurations and Heyting varieties.
8. Opening the second cycle: non-topped configurations and some reasons why they create more trouble.
9. The algorithm for creating first order formulas in the (acyclic) non-topped case.
10. Closing – not quite completely – the second cycle.
11. More on the structure of sums of Priestley spaces:

(a) the tame part, and details of a special case,

(b) creating (new) cycles.

12. Some open problems.

The P-frame reflection of a completely regular frame

Richard N. Ball
University of Denver
rball@du.edu
Coauthors: Joanne Walters-Wayland (University of Denver)

A P-space is one in which every continuous real-valued function is constant in a neighborhood of each point. These spaces are important, since they carry the epicomplete objects in the categories usually chosen to axiomatize $C(X)$, namely archimedean f-rings or archimedean l-groups with weak order unit. Furthermore, there is a P-space coreflection in spaces which is quite close at hand: simply refine the given topology by declaring the new open sets to be countable intersections of the given open sets. However, the localic analog of this coreflection is hardly close at hand, due largely to the fact that the quotient of a P-frame need not be a P-frame. In this talk we will show that the reflection exists nevertheless. Ancillary results which may be of independent interest are that Lindelof degree is preserved by both frame colimits and by the passage to the so-called assembly of a frame, i.e., to the frame of its nuclei.

A result on Complete Hausdorffness in topological algebras

Wolfram Bentz
St. Francis Xavier University
wfbentz@alumni.uwaterloo.ca

In 1977, Walter Taylor showed that T_0-topological algebras in congruence permutable varieties are Hausdorff. This result has been expanded to other classes of varieties by Gumm, Coleman, Kearnes, Sequeira, and the presenter. There have moreover been negative results in support of a particular algebraic characterization; however, such a characterization has not been proofed yet in general.

Utilizing another approach to the above problem, I am currently looking at replacing Hausdorffness with a nominal stronger condition. The original result by Taylor has been extended to complete Hausdorffness by Coleman, a property that states that two distinct points can be separated by open sets having disjoint closures. Further results concerning complete Hausdorffness have appeared in the author's undergraduate thesis and graduate thesis, and suggest that the two topological conditions actually coincide for topological varieties.

In this talk, I will give an introduction to the area of topological universal algebra, introduce the properties in question, and announce a not yet published result. I will also comment on the possibility that Hausdorffness and Complete Hausdorffness are indeed identical under this circumstances, and the difficulties in proofing such a claim.

Free algebras in locally finite semisimple varieties

Joel Berman
University of Illinois at Chicago
jberman@uic.edu

We consider the structure and cardinality of free objects in locally finite semisimple varieties of algebras. We provide upper bounds for the sizes of the finitely generated free algebras in such varieties and present various algebraic consequences that result when these upper bounds are obtained. These techniques are applied to some locally finite semisimple varieties that arise in algebraic logic and we thereby prove some new results and provide alternate proofs for some previously known theorems. Central to this work is the notion of a valuation f from a set X to an algebra $\mathbf{A}$, which is any function f from X to $\mathbf{A}$ such that $f(X)$ generates $\mathbf{A}$. We analyze a variety $\mathcal{V}$ and its free algebra freely generated by a set X by studying the set of all valuations from X into finite algebras $\mathbf{A}$ in $\mathcal{V}$ that are subdirectly irreducible or belong to a set of generators of $\mathcal{V}$.

On the closure of a topology and low separation axioms

Maria Luisa Colasante
Universidad de Los Andes, Mrida, Venezuela
`marucola@ula.ve`
Coauthors: C. Uzctegui and J. Vielma

Let P(X) denote the power set of X. By identifying a set with its characteristic function, we give P(X) the product topology. The closure cl(T) of any topology T on X is the samallest Alexandroff topology containing T ([2]). We present results concerning some low separation axioms that any topology on X shares with T when bounded by T and cl(T). Special consideration is given to the countable case.

References [1] Lo, J.T.H, Borel sets and separtion axioms, Mathematical Proceedings of the Royal Irish Academy, 101A(2)(2001), 111-123. [2] C. Uzctegui and J. Vielam, Alexandroff topologies view as closed sets in the Cantor Cube, Div. Mat. Vol. 13, No.1(2005), 45-53.

On Full Networks and the Galois Connection between Clones and Co-clones

Matthew Cook
Uni-ETH Zurich / Caltech
`cook@ini.phys.ethz.ch`

The classic result from Geiger (1968) and Bodnarchuk et al. (1969) is that the lattices (ordered by set inclusion) of clones (closed sets of functions) and co-clones (closed sets of relations) are identical but flipped. We define a "full network" for a set of available relations, and show how it greatly simplifies the process of testing the implementability of a target relation, and it also greatly simplifies this classic (but lengthy) proof.

Endomorphism Kernel Property in Stone Algebras

Hernando Gaitan
Universidad Nacional de Colombia
hgaitano@unal.edu.co

An algebra has the endomorphism kernel proprty if every congurence on the algebra is the Kernel of one of its endomorphims. We characterize the finite Stone algebras with such a property.

Making the ring of continuous localic real functions into a subring of all localic real functions

Javier Gutiérrez García
Departamento de Matemáticas, Universidad del País Vasco-Euskal Herriko Unibersitatea, Apdo. 644, Bilbao, Spain
javier.gutierrezgarcia@ehu.es
Coauthors: Tomasz Kubiak and Jorge Picado

So far, in pointfree topology, the lattice-ordered ring of all continuous real functions on a frame L has not been a part of the lattice of all lower (or upper) semicontinuous real functions on L [5]. Our goal is to demonstrate a framework in which all those (semi)continuous functions arise (up to isomorphism) as members of the lattice-ordered ring of all frame homomorphisms from the frame of reals into the frame of all congruences on L. That ring is a poinfree counterpart of the ring of *all* real-valued functions on a topological space, thereby providing a pointfree analogue of the concept of an *arbitrary* (not necessarily (semi)continuous) real function on L. One feature of this remarkable conception is that one eventually has: *lower semicontinuous* + *upper semicontinuous* = *continuous*. We document its importance by showing how nicely can the insertion, extension and regularization theorems of [1,2,3] be recast in the new setting.

This talk is a presentation of much of the paper [4].

Bibliography

[1] J. Gutiérrez García, T. Kubiak and J. Picado, Monotone insertion and monotone extension of frame homomorphisms, *J. Pure Appl. Algebra* 212 (2008) 995–968. [2] J. Gutiérrez García, T. Kubiak and J. Picado, Lower and upper regularizations of frame semicontinuous real functions, *Algebra Universalis*, to appear. [3] J. Gutiérrez García, T. Kubiak and J. Picado, Pointfree forms of Dowker's and Michael's insertion theorems, *J. Pure Appl. Algebra*, to appear. [4] J. Gutiérrez García, T. Kubiak and J. Picado, Localic real functions: a general setting, Preprint 08-20 of DMUC, April 2008 (submitted for publication). [5] J. Gutiérrez García and J. Picado, On the algebraic representation of semicontinuity, *J. Pure Appl. Algebra* 210 (2007) 299–306.

Interval-valued residuated lattices

Bart Van Gasse
Ghent University
`Bart.VanGasse@UGent.be`
Coauthors: Chris Cornelis Glad Deschrijver Etienne Kerre

Starting from a bounded lattice $\mathcal{L} = (L, \sqcap, \sqcup, 0, 1)$, one can construct a new bounded lattice (called the triangularization of $\mathcal{L}$) by taking the intervals in $\mathcal{L}$ and defining $[x_1, x_2] \leq [y_1, y_2]$ iff $x_1 \sqcap y_1 = x_1$ and $x_2 \sqcap y_2 = x_2$. If there exists a residuated lattice (RL) [1] on the lattice $\mathcal{L}$, then it is always possible to define residuated lattices on the triangularization of $\mathcal{L}$ such that the set of exact intervals (these are the intervals of the form $[x, x]$) is closed under all four RL-operations (infimum, supremum, the product $*$ and the implication $\Rightarrow$). We call such structures interval-valued residuated lattices (IVRLs) [4].
We will present triangle algebras, which are equationally defined algebraic structures (forming a variety) that are isomorphic to IVRLs.
This representation allows us to show that IVRLs are completely determined by the subalgebra of exact intervals and the value of $[0, 1] * [0, 1]$. Conversely, for every residuated lattice $\mathcal{L} = (L, \sqcap, \sqcup, * \Rightarrow, 0, 1)$ and every a in L, there exists an IVRL in which the subalgebra of exact intervals is isomorphic to $\mathcal{L}$ and in

which $[0,1] * [0,1] = [0,a]$. So there is a one to one correspondence between IVRLs and couples $(\mathcal{L}, a)$ consisting of a RL $\mathcal{L}$ and an element a in that RL [5]. For a number of well-known properties (distributivity, divisibility, involutive negation etc. [2, 3]), we investigate if they can be satisfied on IVRLs, and - if so - under which conditions on $\mathcal{L}$ and a.

Bart Van Gasse and Chris Cornelis would like to thank the Research Foundation–Flanders for funding their research.

Bibliography

[1] R.P. Dilworth and M. Ward Residuated Lattices Trans. A.M.S. 45 (1939), 335–354

[2] F. Esteva, L. Godo, A. Garcia-Cerdaña On the Hierarchy of t-norm Based Residuated Fuzzy Logics in: Beyond Two: Theory and Applications of Multiple Valued Logic (M. Fitting and E. Orlowska, eds.), Physica-Verlag (2003), 251–272

[3] B. Van Gasse, C. Cornelis, G. Deschrijver, E.E. Kerre On the properties of a generalized class of t-norms in interval-valued fuzzy logics New Mathematics and Natural Computation, Vol. 2 (No. 1) (2006), 29–42

[4] B. Van Gasse, C. Cornelis, G. Deschrijver, E.E. Kerre Triangle algebras: A formal logic approach to interval-valued residuated lattices Fuzzy Sets and Systems 159 (2008), 1042–1060

[5] B. Van Gasse, C. Cornelis, G. Deschrijver, E.E. Kerre A characterization of interval-valued residuated lattices To appear in: International Journal of Approximate Reasoning (2008), doi: 10.1016/j.ijar.2008.04.006

More clones from ideals

Martin Goldstern
Technische Universitaet Wien
goldstern@tuwien.ac.at

A clone is a set of functions (of finite arity) over a fixed base set X which contains all projections and is closed under composition (substitution). (Typical examples: the set of term operations of an algebra on X, the set of continuous operations on a topological space with underlying set X.) The set of all clones on X forms a complete lattice, Cl(X).

I am interested in the structure of this lattice, in particular in the co-atoms of Cl(X). They are well-understood for finite X, but difficult to classify for infinite X.

In my talk I will discuss some natural generalizations of constructions for finite sets X; these constructions often replace a single set by a filter or ideal of sets.

Permutation Models and Symmetric Models

Eric Hall
UMKC
`halle@umkc.edu`

A "permutation model" is a model of ZFA (ZF modified to allow atoms) used to give independence results regarding AC and related propositions (also known as a Fraenkel-Mostowski model). A symmetric model, used to produce independence results in ZF (without atoms), can be obtained by forcing over a permutation model, though this is not the usual characterization.

Limit-like predictability for discontinuous functions

Christopher S. Hardin
Smith College
`chardin@email.smith.edu`
Coauthors: Alan D. Taylor (Union College)

Our starting point is the following question: To what extent is a function's value at a point x of a topological space determined by its values in an arbitrarily small (deleted) neighborhood of x? For continuous functions, the answer is typically "always" and the method of prediction of $f(x)$ is just the limit operator. We generalize this to the case of an arbitrary function mapping a topological space to an arbitrary set. We show that the best one can ever hope to do is to predict correctly except on a scattered set. Moreover, we generalize the mu-strategy from [1] to obtain a predictor whose error set, in T_0 spaces, is always scattered.

The techniques are carried out in structures, which we call proximity schemes, that generalize both topological spaces and binary relations. A proximity scheme associates each point in a set X with a filter on X. To describe a topological space X, we associate each point with the filter generated by its neighborhoods. To describe a binary relation R on X, we associate each point x with the principal filter generated by the set of y such that xRy.

References:

[1] Christopher S. Hardin and Alan D. Taylor. A peculiar connection between the axiom of choice and predicting the future. *American Mathematical Monthly*, 115(2):91-96, February 2008.

Orthomodularity in dagger biproduct categories

John Harding
New Mexico State University
jharding@nmsu.edu

Abramsky and Coecke recently introduced an approach to finite dimensional quantum mechanics based on strongly compact closed categories with biproducts. We show that the projections of any object A in such a category forms an orthoalgebra $Proj\ A$. Sufficient conditions are given to ensure this orthoalgebra is an orthomodular poset. A notion of a preparation for such an object is given by Abramsky and Coecke, and it is shown that each preparation induces a finitely additive map from $Proj\ A$ to the unit interval of the semiring of scalars for this category. The tensor product for the category is shown to induce an orthoalgebra bimorphism $ProjA \times ProjB \to Proj(A \otimes B)$ that shares some of the properties required of a tensor product of orthoalgebras.

These results are established in a setting more general than that of strongly compact closed categories. Many are valid in dagger biproduct categories, others require also a symmetric monoidal tensor compatible with the dagger and biproduct structure. Examples are considered for several familiar strongly compact closed categories.

Sheaf Locales

Wei He
Dep.Mathematics,Nanjing Normal University
weihe@njnu.edu.cn
Coauthors: Maokang Luo

Let X be a locale. The equivalence of the category $Sh(X)$ of sheaves on X and the slice category LH/X of local homeomorphisms over X has already known, as is implicit in Johnstone [1]. The purpose of this paper is to give an explicit construction of associated sheaf locales, and prove directly the equivalence of the category $Sh(X)$ and LH/X.

Top and Bottom Varieties of Pseudo-MV Algebras

W. Charles Holland
University of Colorado
`Charles.Holland@Colorado.edu`

Pseudo-MV-algebras are the non-commutative generalization of MV-algebras, which are the algebras associated with Lukasiewicz multi-valued logic. They are closely associated with lattice-ordered groups. These subjects arose in the early twentieth century inspired, partly, by problems arising from quantum mechanics.

Varieties of these algebras are equationally defined classes. The lattice of varieties of pseudo-MV-algebras is essentially identical to the lattice of varieties of lattice-ordered groups with a specified strong unit. The lattice of varieties of (commutative) MV-algebras was completely described by Komori (1981). Much, but not all, is known about the lattice of varieties of lattice-ordered groups. Varieties of the non-commutative pseudo-MV-algebras have only recently been investigated. I will describe those varieties which are"near the top" and those "near the bottom" of the lattice of all varieties of pseudo-MV-algebras.

This is on-going work with A. Dvurečenskij, A. Glass, and M. Darnel.

References

1. Holland, W. C., *Small varieties of lattice-ordered groups and MV-algebras*, in Contributions to General Algebra **16**(2005), 107–114, Heyn, Klagenfurt.

2. Holland, W. C., *Covers of the Boolean variety in the lattice of varieties of unital lattice-ordered groups and GMV-algebras*, Izvrannie Voprosi Algebra (2007), Memorial Volume for N. Ya. Medvedev, 208–217.

3. Holland, W. C., and Dvurečenskij, A., *Top varieties of generalized MV-algebras and unital lattice-ordered groups*, Comm. Algebra **35**(2007), no. 11, 3370–3390..

4. Holland, W. C., and Dvurečenskij, A., *Komori's characterization and top varieties of GMV-algebras*, (to appear in Algebra Universalis).

5. Holland, W. C., and Dvurečenskij, A., *Covers of the Abelian variety of generalized MV-algebras* , (in progress).

6. Holland, W. C., and Glass, A., *Top periodic varieties of unital lattice-ordered groups*, (in progress).

7. Holland, W. C., and Darnel, M., *Bottom varieties of unital lattice-ordered gro*

Minimal Varieties of Cancellative Residuated Lattices

Rostislav Horcik
Institute of Computer Science, Academy of Sciences of the Czech Republic
horcik@cs.cas.cz

The main goal of our talk is to investigate the varieties of cancellative residuated lattices covering the variety $\mathcal{V}(\mathbb{Z}^-)$ generated by the negative cone of integers. We consider the varieties generated by finitely generated submonoids of $\mathbb{Z}^-$ (note that each such submonoid can be enriched by a residuum so that it forms a cancellative residuated lattice) and show that some of them generate covers of $\mathcal{V}(\mathbb{Z}^-)$. Namely, we prove that each submonoid of $\mathbb{Z}^-$ generated by two coprime numbers such that the greater one is prime generates a cover of $\mathcal{V}(\mathbb{Z}^-)$. Moreover, we can show that two different pairs of coprime numbers such that the greater one is prime generate different covers of $\mathcal{V}(\mathbb{Z}^-)$. Consequently, it follows that there are infinitely many covers of $\mathcal{V}(\mathbb{Z}^-)$.

The comparison of various club guessing principles.

Tetsuya Ishiu
Miami University
`ishiut@muohio.edu`
Coauthors: Justin Moore, Paul Larson

I will present some of the variations of the club guessing principle, and outline the proofs that there are no non-trivial implications between them. In particular, I will explain how to prove appropriate iteration theorems.

Ordered algebraic structures in the open-source mathematics system Sage

Peter Jipsen
Chapman University
`jipsen@chapman.edu`

Several computer algebra systems, such as Maple and Mathematica, have packages for doing calculations with posets and lattices. Recently the free open-source mathematics system Sage (sagemath.org) has also acquired some built-in support for such calculations. I will demonstrate some extensions of these features to several classes of ordered algebraic structures, including residuated lattices, idempotent semirings (with domain operator), modal lattices, and allegories. This aids with illustrating a structure theorem for n-potent divisible residuated lattices as Heyting products of MV-chains. Extensions of this result to some classes of idempotent semirings, distributive modal lattices and distributive allegories will also be discussed.

From finite topologies and lattices to the automorphism groups of rational Schur rings

Mikhail Klin
Ben-Gurion University of the Negev, Beer Sheva, Israel
`klin@cs.bgu.ac.il`
Coauthors: Istvan Kovacs

A Cayley graph Γ *over cyclic group* $\mathbb{Z}_n$ *of order* n *is called a circulant graph. Graph* Γ *is called rational if the spectrum of its adjacency matrix consists only of rational (in fact integer) numbers. The main motivation of this presentation stems from our interest to establish efficient procedure to determine automorphism group of a rational circulant graph.*

We employ methodology of Schur rings (briefly, S-rings), see [3].

Problem of classification of rational S-rings (S-rings of traces in original Schur-Wielandt's terminology) goes back to the seminal paper of Schur (1933). Various particular cases were considered a few decades ago by Ya. Yu. Gol'fand, Klin and R. Pöschel. In particular, for the case when n *is a product of* k *distinct primes, Gol'fand (1985) established a bijection between rational S-rings over* $\mathbb{Z}_n$ *and finite topologies on the set of cardinality* k.

An elegant description of all rational S-rings over $\mathbb{Z}_n$ *was provided by Muzychuk [2] in terms of sublattices of the lattice of all natural divisors of* n.

Another origin of our approach stems to the operation of crested product of association schemes [1]. Note that similar concepts for the particular case of S-rings over cyclic groups were suggested by K. H. Leung and S. C. Ma, S. Evdokimov and I. N. Ponomarenko. A more general operation of wedge product of association schemes is treated by Muzychuk.

Merging ideas of our predecessors, we consider simple reduction rules which allow for a given rational S-ring S *over* $\mathbb{Z}_n$ *to describe its automorphism group* $Aut(\Gamma)$. *Special attention is paid to those particular cases when* $Aut(\Gamma)$ *appears via iterative use of wreath products and direct products of symmetric groups. For a rational circulant graph* Γ *we determine such S-ring* S *that* $Aut(\Gamma) = Aut(S)$.

Note that it follows from our results that all rational S-rings over $\mathbb{Z}_n$ *are Schurian.*

This expository talk is based on the cooperation of the authors with O.H. Kegel and M. Muzychuk.

References

[1] Bailey, R. A.; Cameron, Peter J. Crested products of association schemes. J. London Math. Soc. (2) 72 (2005), no. 1, 1–24. [2] Muzychuk, Mikhail E. The structure of rational Schur rings over cyclic groups. European J. Combin. 14 (1993), no. 5, 479–490. [3] Muzychuk, Mikhail; Klin, Mikhail; Pöschel, Reinhard. The isomorphism problem for circulant graphs via Schur ring theory. Codes and association schemes (Piscataway, NJ, 1999), 241–264, DIMACS Ser. Discrete Math. Theoret. Comput. Sci., 56, Amer. Math. Soc., Providence, RI, 2001.

Probability Metrics and their applications

Shaghayegh Kordnourie
Department of statistics ,The Islamic Azad University North Tehran Branch;Iran
`shaghayeghkordnourie@yahoo.com`
Coauthors: Hamidreza Mostafaei Department of statistics ,The Islamic Azad University North Tehran Branch;Iran E-mail:hrmostafaei@yahoo.com

Abstract. In this article, we introduce the definitions and characteristics of some important probability metrics (distances); furthermore, we examine some applications of these metrics. The relationships among these metrics are evaluated. Finally, we study the convexity property of metrics and this property is investigated for the Birnbaum-Orlicz average distance. Key words: probability metrics, Relative entropy,Wasserstein metric, Birnbaum-Orlicz average distance, convexity

A finite basis problem in loop theory

Tomasz Kowalski
University of Cagliari
`kowatomasz@gmail.com`

A loop can be thought of as a group-like object, with a nonassociative multiplication. A loop L is a group if and only if L is associative. One effect of nonassociativity of L is that powers of an element a are not well-defined
(a^3isambiguousbetween$(a*a)*a$anda$*(a*a)$, say).Sonoteveryone−generatedsubloopofLisagroup.Conside
generatedsubloopisagroup.Suchloopsarecalledpower−associative, andformanon−
finitelybasedvariety, asshownbyEvansandNeumannin[1].Theyalsoconsidereddiassociativeloops :
loopssuchthateverytwo−generatedsubloopisagroup, andaskedwhetherthesearefinitelybased.Anegativeso

We will show that diassociativity is not finitely based even relative to power associativity. Our result is stronger than the one announced in [4] and the proof technique is different: we use elementary constructions and a standard ultraproduct argument.

References

[1] T. Evans, B.H. Neumann, On varieties of groupoids and loops, J. London Math. Soc. 28 (1953) 342-350.

[2] D.M. Clark, Diassociative groupoids are not finitely based, J. Australian Math. Soc. 11 (1970) 113-114.

[3] M.K. Kinyon, K. Kunen, J.D. Phillips, A generalisation of Moufang and Steiner loops, Algebra Universalis 48, 1 (2002) 81-101.

[4] W.D. Smith, Loop diassociativity has no finite basis, unpublished manuscript (2005).

The arity gap of finite functions and generalizations of Świerczkowski's lemma

Erkko Lehtonen
University of Waterloo, Tampere University of Technology
erkko.lehtonen@tut.fi
Coauthors: Miguel Couceiro (University of Luxembourg)

The arity gap *of a function $f\colon A^n \to B$ of several variables is defined as the minimum decrease in the number of essential variables when pairs of essential variables of f are identified. The study of this notion goes back to the 1963 paper by Salomaa [4] who showed that the arity gap of any Boolean function is at most* 2*. This called for a classification of Boolean functions according to their arity gap, which we established in [1]. Willard [6] extended Salomaa's result to functions on arbitrary finite domains and codomains by showing that the same upper bound holds for the arity gap of any function $f : A^n \to B$ with A and B finite, provided that $n > |A|$.*

To complement these previous results, we determine the arity gap of functions $f\colon A^n \to B$ with a small number of essential variables, i.e., when $n \leq |A|$, and we obtain a classification of functions according to their arity gap in terms of so-called quasi-arity and determinability by oddsupp *(see [2]). Furthermore, an explicit classification of pseudo-Boolean functions according to their arity gap is obtained.*

We also discuss how our results on the arity gap can be used to generalize Świerczkowski's lemma [5], which essentially asserts that given an operation $f\colon A^n \to A$, if every operation obtained from f by identifying a pair of variables is a projection, then f is a semiprojection, i.e., $f(a_1, \ldots, a_n) = a_t$ for some $1 \leq t \leq n$ whenever $a_i = a_j$ for some $1 \leq i < j \leq n$.

References

[1] M. Couceiro, E. Lehtonen, *On the effect of variable identification on the essential arity of functions on finite sets, Int. J. Found. Comput. Sci.* **18** *(2007) 975–986.*

[2] M. Couceiro, E. Lehtonen, *Generalizations of Świerczkowski's lemma and the arity gap of finite functions, arXiv:0712.1753v1.*

[3] M. Couceiro, M. Pouzet, *On a quasi-ordering on Boolean functions, Theoret. Comput. Sci. (2008), doi:10.1016/j.tcs.2008.01.025.*

[4] A. Salomaa, *On essential variables of functions, especially in the algebra of logic, Ann. Acad. Sci. Fenn. Ser. A I. Math.* **339** *(1963) 3–11.*

[5] S. Świerczkowski, *Algebras which are independently generated by every n elements, Fund. Math.* **49** *(1960) 93–104.*

[6] R. Willard, *Essential arities of term operations in finite algebras, Discrete Math.* **149** *(1996) 239–259.*

Topological Semantics with Settling

Robert Lubarsky
Florida Atlantic University
Robert.Lubarsky@comcast.net

Under the standard topological semantics, sets are thought of as growing as the opens of the topological space shrink. With settling, the (currently extant) sets can at any time become fixed as (images of) ground model sets, and hence stop growing as the opens shrink, while the variable sets become re-incarnated around them. The model of set theory so generated satisfies IZF, save only for Power Set becoming weakened. This is useful for proving some independence results. Other variants of topological models will also be mentioned.

"Iff" is not expressible in independence-friendly logic

Allen L. Mann
Colgate University
allen.l.mann@gmail.com

Ordinary first-order logic has the property that two formulas ϕ and ψ have the same meaning in a structure exactly when the formula "ϕ if and only if ψ" is true in the structure. We prove that independence-friendly logic does not have this property.

An algebraic generalization of Kripke structures

Sergio Marcelino
King's College London
sergiortm@gmail.com
Coauthors: Pedro Resende

In [1] the Kripke semantics of classical propositional normal logic is made algebraic via an embedding of Kripke structures into the larger class of pointed stably supported quantales. This algebraic semantics subsumes the traditional algebraic semantics based on lattices with unary operators, and it suggests natural interpretations of modal logic, of possible interest in the applications, in structures that arise in geometry and analysis, such as foliated manifolds and operator algebras, via topological groupoids and inverse semigroups. In this talk I survey some results and examples of [1].

[1] S. Marcelino, P Resende, An algebraic generalization of Kripke structures to appear in Math. Proc. Cambridge Philos. Soc., http://arxiv.org/abs/0704.1886

Constraint Satisfaction Problem for algebras generating a congruence-distributive variety

Petar Markovic
University of Novi Sad, Serbia
pera@im.ns.ac.yu
Coauthors: Catarina Carvalho, Victor Dalmau and Miklos Maroti

I will present some ideas which have been used in the proofs of bounded width for CSP with template invariant under a short sequence of Jonsson terms (so-called CD(3) and CD(4) cases) and some new ideas arising from Barto, Kozik and Niven's result on dichotomy of CSP for directed graphs without sources and sinks.

Structural Completeness for Fuzzy Logics

George Metcalfe
Vanderbilt University
george.metcalfe@vanderbilt.edu
Coauthors: Petr Cintula

A consequence relation is structurally complete iff all of its proper extensions have new theorems. That is, if a schematic rule is admissible (preserves the set of theorems), then it is derivable in any formal system axiomatizing the consequence relation. Classical Logic has this property, but for many families of non-classical logics, intermediate, modal, and substructural, it is relatively rare, often holding only for fragments of the systems in question. We investigate structural completeness for the class of (t-norm based) fuzzy logics which includes Godel Logic, known to be structurally complete, and Lukasiewicz Logic, where the positive fragment is known to have the property, but not the full logic. Using a range of methods, we are able to obtain an almost complete picture of structural completeness for this family. In particular, we show that structural completeness for these systems is characterized by the existence of an embedding of the generators of the class of algebras for the logic into the free algebra for this class, and use this observation to obtain structural completeness for Product Logic and fragments of Basic Logic.

Superproducts of Lattices, Boolean and DeMorgan algebras

Yu. M. Movsisyan
Yerevan State University
yurimovsisyan@yahoo.com
Coauthors: A.B. Romanowska (Warsaw University of Technology), J.D.H. Smith (Iowa State University)

We consider the category of algebras with bihomomorphisms $(\varphi, \tilde{\psi})$ as morphisms, where:

$$\varphi\left[A(x_1, \ldots, x_n)\right] = (\tilde{\psi} A)\,(\varphi x_1, \ldots, \varphi x_n)\,.$$

This category we denote by $\widetilde{Alg}$. Product in this category is called a superproduct of algebras.

We describe superproducts of lattices, modular lattices, distributive lattices, Boolean and DeMorgan algebras. In particular, we get the Ginsberg-Fitting theorem for bounded bilattices in logic programming and Tarski-Yershov theorem on solvability of elementary theory of Boolean algebras.

A Stone-type duality for Meet Semilattices

Aditya Nagrath
University of Denver
`adnagrat@math.du.edu`

We generalize Priestley duality to arrive at a duality for an interesting category of bounded meet semilattices with two natural properties. The dual objects of the meet semilattices are a category of topological spaces endowed with a ternary relation called betweenness. As with Priestley duality, our duality results from what is known categorically as a full duality between the two categories in question.

Some properties of the supersoluble formation and the supersoluble residual of a group

Hassan Naraghi
Islamic Azad University, Ashtian Branch, Iran.
`hasan_naraghi@yahoo.com`

Let p,q,r be primes such that pq is not divisor of r-1 and $p < q < r$*. We say that the subgroups H and K of a group G are mutually permutable if H permutes with every subgroup of K and K permutes with every subgroup of H. If G=HK and K are mutually permutable, we say that G is the mutually permutable product of the subgroups H and K.*
It is known that the class $\mathfrak{U}$ *of all finite supersoluble group is a formation. This means that if a finite group G is supersoluble and N is a normal subgroup of G, then* G/N *is supersoluble and if M and N are two norrmal subgroups of a finite group G, then* $G/(M \cap N)$ *is supersoluble provided that* G/M *and* G/N *are supersoluble. Consequently, every finite group G has a smallest normal subgroup with a supersoluble quotient. This subgroup is called supersoluble residual of G and it is denoted by* $G^{\mathfrak{U}}$*. It is clear that* $G^{\mathfrak{U}}$ *is epimorphism-invariant and so it is a characteristic subgroup of G(see[2;II,2.4]).*
This paper focuses attention on the study of supersoluble subgroups and supersoluble residual of the group G=[W][V]X to semidirect product and consider the subgroups H=[W]X and K=[W]V of G such that X is the cyclic group of order p, and V is an irreducible and faithful X-module over GF(q), and Y=[V]X is the corresponding semidirect product and W is an irreducible and faithful Y-module over GF(r). we determine that G is the mutually permutable product of subgroups H and K. Moreover, H is not a supersoluble subgroup of G. On the other hand, $K \in \mathfrak{U}$*. Moreover* $H^{\mathfrak{U}} < W$*. However* $G^{\mathfrak{U}} = W$*.*

Tutorial in Set-Theoretic Topology

Peter Nyikos
University of South Carolina
nyikos@math.sc.edu

This tutorial is designed to give a feel for the techniques and topics in set-theoretic topology through the presentation of some standard tools and examples. The axiom of choice (in the forms of Zorn's Lemma and well-ordering) will be used to construct some examples, while others will require the use of axioms independent of the usual ones, such as the continuum hypothesis (CH) and the combinatorial principles $\clubsuit$ and $\diamondsuit$. These constructions in turn will be contrasted with theorems using such tools as Martin's Axiom (MA) together with the negation of CH.

Most of the topological properties involved will be quite easy to define. In fact, of the charms of set-theoretic topology is that it settles a great many relationships between fundamental topological concepts. For instance, take the following simple question: is every perfectly normal, countably compact space compact? [A perfectly normal space is a normal—this includes Hausdorff—space in which every closed set is a countable intersection of open sets.] The axiom $\diamondsuit$ can be used to construct a counterexample. On the other hand, MA + not-CH implies that every perfectly normal, countably compact space is compact. It is also noteworthy that even CH is insufficient to settle this question either way: on the one hand, $\diamondsuit$ implies CH; on the other hand, there is a model of CH in which every perfectly normal, countably compact space is compact.

Some varieties of residuated lattices with countably many subvarieties

Jeffrey S. Olson
Norwich University
jolson@norwich.edu

The variety of residuated lattices (in the sense established by C. Tsinakis and his collaborators) contains uncountably many subvarieties. More than this, many of its known subvarieties have uncountably many subvarieties, even in cases where considerable structural conditions have been imposed. For example, the variety of residuated lattices which are idempotent *(i.e., which satisfy $x \cdot x \approx x$) and* semilinear *(i.e., subdirect products of linearly ordered residuated lattices) has uncountably many subvarieties, as was shown by N. Galatos. The proof relies on algebras whose monoid operation is non-commmutative. It is of interest then, to determine if the variety of idempotent semilinear residuated lattices which are commutative (*SLIC *for short) has uncountably many subvarieties.* SLIC *has received attention lately, in part because it provides an algebraic semantics for an important extension of a fragment of linear logic.*

We present a construction for residuated lattices which have a linearly ordered lattice reduct and a commutative monoid operation (not necessarily idempotent), which offers some transparency with respect to characterizations of subalgebras and homomorphic images. The finitely generated subdirectly irreducible members of the varieties we investigate may be produced by applying this construction to simple *members of the variety.* SLIC *is generated in this fashion. We provide natural conditions on a class of simple algebras which guarantee that the generated variety has a* countable *number of subvarieties. This allows us to identify many varieties of residuated lattices with a countably infinite number of subvarieties. In particular,* SLIC *is among them.*

On ordered structure of propositional models

Chihiro Oshima
Texas AM INternational University
coshima@tamiu.edu

We introduce an order in propositional models and explore lattice structures that characterize propositional classes. Among these results, we obtain that a strict Horn class forms an algebraic lattice and that a non-strict Horn class forms an algebraic chopped lattice. We also prove that a (chopped) algebraic lattice is isomorphic to a (non)-strict Horn class.

Insertion in spaces, bispaces, ordered spaces and point-free spaces

Jorge Picado
University of Coimbra, Portugal
picado@mat.uc.pt
Coauthors: Maria Joo Ferreira and Javier Gutirrez Garca

Insertion theorems on the existence of continuous real functions separating lower and upper semicontinuous real functions (like the Katětov-Tong insertion theorem characterizing normal spaces and the Stone insertion theorem characterizing extremally disconnected spaces) rank among the fundamental results in point-set topology. In this talk we present results of that kind for biframes (the point-free counterpart of bitopological spaces) that generalize at once the classical result and its counterparts in bitopological spaces (Priestley [6]), ordered topological spaces (Nachbin [4], Priestley [6]) and frames (recently obtained in [5], [3] and [1]; see also [2]).

References

[1] J. Gutirrez Garca, T. Kubiak and J. Picado, Lower and upper regularizations of frame semicontinuous real functions, Algebra Universalis*, to appear.*

[2] J. Gutirrez Garca, T. Kubiak, and J. Picado, Localic real-valued functions: a general setting*, Preprint 08-20 of DMUC, April 2008 (submitted for publication).*

[3] J. Gutirrez Garca and J. Picado, On the algebraic representation of semicontinuity, J. Pure Appl. Algebra *210 (2007) 299–306.*

[4] L. Nachbin, Topology and Order*, Van Nostrand Mathematical Studies 4, Princeton, 1965.*

[5] J. Picado, A new look at localic interpolation theorems, Topology Appl. *153 (2006), 3203–3218.*

[6] H. Priestley, Separation theorems for semi-continuous functions on normally ordered topological spaces, J. London Math. Soc. *(2) 3 (1971) 371-377.*

Workshop on forbidden configurations in Priestley spaces

Ales Pultr
Charles University
pultr@kam.mff.cuni.cz
Coauthors: Richard N. Ball (University of Denver)

1. *A brief review of Priestley duality, including the full-blown version.*

2. *Configurations. Forbidden configurations and resulting classes of distributive lattices. A few classical results and problem setting.*

3. *Generating first-order formulas by forbidden trees.*

4. *First negative result: the diamond.*

5. *Coproducts of Priestley spaces. Their existence and general form. The coproductivity problem.*

6. *Los' Theorem and closing the first cycle:*

For a topped configuration P the following statements are equivalent:

- *P is acyclic,*
- *P is coproductive,*
- *forbidding P in $\mathcal{P}(L)$ creates a first-order class of distributive lattices.*

7. *Aside: Prohibiting configurations and Heyting varieties.*

8. *Opening the second cycle: non-topped configurations and some reasons why they create more trouble.*

9. *The algorithm for creating first order formulas in the (acyclic) non-topped case.*

10. *Closing – not quite completely – the second cycle.*

11. *More on the structure of sums of Priestley spaces:*

(a) the tame part, and details of a special case,

(b) creating (new) cycles.

12. *Some open problems.*

Algebra of Logic (tutorials)

James Raftery
University of KwaZulu-Natal
raftery@ukzn.ac.za

The tutorials will touch, in rather selective depth, on the following topics:

- Aspects of the subject's history, including the calculus of binary relations; relation algebras; residuation.

- The arrival of non-classical logics in the twentieth century; various substructural logics.

- The arrival of relational semantics for non-classical logics.

- The role of algebraic methods in relational semantics: modal logic as a case study.

- The arrival of new universal algebraic methods from the 1960s onward; striking applications: degrees of incompleteness.

- Abstract consequence relations.

- The equivalence of consequence relations; algebraizable logics.

- Some 'bridge' theorems between algebra and logic.

- Matrix theory and the Leibniz operator.

- *Classification of deductive systems; the Leibniz hierarchy.*

Paracompact box products

Judith Roitman
University of Kansas
roitman@math.ku.edu

The question of which box products are paracompact is perhaps the last of the great set theoretic topology problems from the 1950's without a satisfactory resolution. This talk is a survey of the problem, from the 1970's to the present.

The last forcing standing

Andrzej Roslanowski
University of Nebraska at Omaha, Department of Mathematics, Omaha NE68182
roslanow@member.ams.org
Coauthors: Saharon Shelah

In a sequence of previous papers ([RoSh:655], [RoSh:777], [RoSh:860], [RoSh:888] and [RoSh:890]) we introduced several properties of forcing notions guaranteeing that their λ–support iterations a proper (for an uncountable cardinal λ). Several forcing notions built according to a scheme somewhat similar to forcings with tress and creatures *were covered by those properties, however one example of interest to us was not included. In the talk we will introduce a property and an iteration theorem showing that for an inaccessible cardinal λ, one may succesfully iterate with λ–support the following forcing notion Q_λ:*

Conditions in Q_λ are complete perfect trees $T \subseteq {}^{<\lambda}\lambda$ in which every splitting is to a club of λ and the limit of an increasing sequence of splitting nodes is a spliting node. The order is the inclusion.

Fuzzy bitopological spaces via fuzzy ideals

A.A. Salama
egypt
ahmed_salama_2000@yahoo.com

In this paper we introduce the notion of fuzzy bitopological ideals. The concept of fuzzy pairwise local function is also introduced here by utilizing the q-neighborhood structure for a fuzzy topological space. These concepts are discussed fuzzy bitopologies and several relations between different fuzzy bitopological ideals.

Localization Operators in Quantum Mechanics on Phase Space as an M.V. Algebra and a Heyting Algebra

Franklin E. Schroeck, Jr.
University of Denver
`fschroec@du.edu`

In quantum mechanics certain localization operators arise naturally in the phase space (symplectic space) formalism. We will show how to derive them and show that they form an M.V. algebra and a Heyting algebra with a different complementation.

Commutator relations and the clones of finite groups

Jason Shaw
University of Colorado
`jason.shaw@colorado.edu`

If G is a group, then the clone of G, denoted $Clo(G)$, is the set of finitary operations on G which are the interpretations of group words. It is of interest to ask: Does there exists an integer $k > 0$ such that for all groups G, $Clo(G)$ is determined by the subgroups of G^k? By determined we mean that an operation f is in $Clo(G)$ if and only if f preserves the subgroups of G^k. In this presentation we show that there does not exist $k > 0$ such that for all finite groups G, $Clo(G)$ is determined by the subgroups of G^k.

On categories of ordered sets with a closure operator

Josef Slapal
Brno University of Technology
`slapal@fme.vutbr.cz`

We define and study two categories of partially ordered sets endowed with a closure operator. The first category has order-preserving continuous maps as morphisms and it is shown to be concretely isomorphic to a category of ordered sets endowed with a compatible preorder. The second category studied has closed maps as morphisms and it is proved to be cartesian closed. As examples, consequences of these results for categories of the usual closure spaces and, in particular, of topological spaces are discussed.

The Strong Condition on Inherently Nonfinitely Based Algebras

Zoltan Szekely
University of Guam
`zoltan_szekely@yahoo.com`

A finite algebra of finite type is inherently nonfinitely based provided it is not a member of any locally finite finitely based variety. A finite algebra A of finite type is said to be constantly bounded by the constant c if any finite algebra B is in the variety generated by A if and only if B satisfies those axioms of A that are no longer than c. It can be shown that if A is constantly bounded then it is either finitely based or inherently nonfinitely based. We say that A satisfies the strong condition on inherently nonfinitely based algebras if A is constantly bounded but not finitely based. We investigate the distinction between inherently nonfinitely based finite algebras and finite algebras with the strong condition on inherently nonfinitely based algebras (shortly finite algebras with the strong condition).

Infinite Hat Problems

Alan D. Taylor
Union College
`taylora@union.edu`
Coauthors: Christopher S. Hardin

In general, a hat problem involves a collection of players who are to have hats of various colors placed on their heads. Typically, each player can see the hats worn by the others, but not his own. The goal is to find a strategy that will ensure correct (independent) guesses by most (e.g., all but finitely many) of the players. We concentrate here on the case in which the set of players is an ordered set and each can see only the hats of higher numbered players. One instance of this is our result that appeared in the February 2008 issue of the Monthly and which asserts that if time is modeled by the real line, then the state of any system at time t is completely determined by its past states for all but a countable, nowhere dense set of instants t.

Approximate satisfaction of identities

Walter Taylor
University of Colorado
`walter.taylor@colorado.edu`

For a topological algebra $\mathbf{A}$ based on a metric space $A = (A, d)$, and for Σ a set of equations in the appropriate language, we define $\lambda_{\mathbf{A}}(\Sigma)$ to be the sup of $d(\sigma^{\mathbf{A}}(a), \tau^{\mathbf{A}}(a))$, over all $\sigma \approx \tau \in \Sigma$ and over all appropriate a. Then, for a metric space A, $\lambda_A(\Sigma)$ is defined as the inf of $\lambda_{\mathbf{A}}(\Sigma)$, over all topological algebras on A. $\lambda_A(\Sigma)$ measures how far the equations Σ must deviate from being satisfied on A with continuous operations. It is zero if Σ is so satisfiable, but may also be zero in the contrary case, even for compact A. This is an ongoing work on elementary calculations of, and facts about, λ.

We know λ_A completely for A an n-sphere (except for $n = 3$ and $n = 7$): $\lambda_A(\Sigma)$ is either 0 or the diameter of S^n. The rest of our knowledge is sporadic. λ_A is not a topological invariant of A, but depends heavily on the metric of A. Except that, for A compact, the condition $\lambda_A(\Sigma) \neq 0$ is a topological invariant (in which case there may or may not be a uniform bound of $\lambda_A(\Sigma)/diam(A)$ away from zero).

For certain A, such as a closed unit interval, we have: if $\lambda_A(\Gamma \times \Delta) < \varepsilon$, then $\lambda_A(\Gamma) < 4\varepsilon$ or $\lambda_A(\Delta) < 4\varepsilon$. (Here $\times$ denotes the varietal product (the meet in the interpretability lattice).) This is an analog, for approximate satisfaction, to the known result, for topological algebras, that if $[0,1]$ models $\Gamma \times \Delta$, then $[0,1]$ models Γ or $[0,1]$ models Δ.

Let A be the realization of a finite simplicial complex, and let α be a computable real number. By simplicial approximation, the set of all finite Σ (in a fixed alphabet) such that $\lambda_A(\Sigma) < \alpha$ is recursively enumerable.

Aspects of set theory with applications to relational structures

Katie Thompson
KGRC, University of Vienna
`thompson@logic.univie.ac.at`

We will start out with a brief history of the foundations of set theory including the axioms of ZFC and models of set theory. A somewhat detailed explanation will be give on how to extend a model of set theory using forcing and contrast the outer models obtained in this way with inner models.

Why are we interested in expanding and shrinking models of set theory? The answer is decidability vs. undecidability. We can use forcing to show that certain mathematical statements cannot be decided (to be true or false) using only models a given axiomatic framework. What happens then if we add to our axioms? How strong does an axiom need to be to decide such statements? The notion of "strength" in this sense is a measure of how many questions an axiom can decide. Also, examples will be given where such axioms and other set theoretic assumptions were used as stepping stones or test cases in proving important mathematical theorems.

To give some flavor of how various aspects of set theory can be applied, we focus on the classification of relational structures. In particular, I am interested in sets with a single binary relation (with possible topological extensions), e.g. linear orders, partial orders, trees, graphs, directed graphs, boolean algebras. Some of these structures have been studied and classified to a certain extent using model theory. However, where model theoretic techniques fail, we can apply set theory to answer questions. The techniques used include infinite combinatorics, forcing and adding in axioms to decide questions.

Time permitting, we will also discuss an application for inner models and how this gives uniformized limitations to the power of forcing.

Ordered Groups in Logic

Constantine Tsinakis
Vanderbilt University
`constantine.tsinakis@vanderbilt.edu`

We propose a new paradigm for the study of various classes of residuated algebras (algebraic counterparts of propositional substructural logics) by viewing them as lattice-ordered groups with a suitable modal operator. This analysis makes precise the view that some of the most interesting algebras arising in algebraic logic are intimately related to lattice-ordered groups.

Alexandroff topologies and semirings

Jorge Vielma
Universidad de los Andes
`vielma@ula.ve`
Coauthors: A Pea and L Ruza

In this work by a semiring R we understand a commutative semiring with identity. Spec(R) denotes the set of all prime ideals of R, equiped with the Zariski topology tz

We prove that (Spec(R),tz) is Alexandroff if and only if R is a Gilmer semiring.

References

[1] Alarcon F and Anderson D, Commutative semirings and their lattices of ideals, Houston Journal of Mathematics. 20 (4) (1994), 571-590.

[2] Golan J. S, Semirings and their applications, Kluwer Academic Publishers, Dordrecht (1999).

[3] Uzcategui, C. and Vielma, J.,Alexandroff Topologies viewed as closed subsets of the Cantor cube, Divulgaciones Matematicas. 13 (1)(2005),45-53

Implication algebras as hypergraphs

Petr Vojtechovsky
University of Denver
`petr@math.du.edu`
Coauthors: Richard N. Ball and J. D. Phillips

Implication algebras are in one-to-one correspondence with semi-boolean algebras. After explaining how every semi-boolean algebra is an upper section of a boolean algebra, we show a correspondence between finite implication algebras and certain hypergraphs. Counting techniques from graph theory will then allow us to count implication algebras of small height.

We will also look at the analogous problem for quasi-implication algebras and orthomodular lattices.

Representation of topological modal algebras

Jacob Vosmaer
University of Amsterdam
`J.Vosmaer@uva.nl`

We want to state a duality theorem for a category of topological modal algebras. So first we may ask: what is a topological algebra? Algebras (in the sense of universal algebra [BurSan2000]) are defined as structures consisting of sets and functions. One could thus topologize algebras by replacing 'sets' with 'topological spaces' and 'functions' with 'continuous functions'.

Definition Let $\langle A; (f_i)_{i\in I}\rangle$ be an algebra, and let $\langle A, \tau\rangle$ be a topological space. We say that $\langle A; (f_i)_{i\in I}, \tau\rangle$ is a *topological algebra* if $f_i \colon A^{n_i} \to A$ is continuous for the product topology on A^{n_i}. A *continuous homomorphism* $f\colon \langle A; (f_i)_{i\in I}, \tau_A\rangle \to \langle B, (g_i)_{i\in I}, \tau_B\rangle$ is a function that is both an algebra homomorphism and a continuous function.

In this abstract we will only consider topological algebras with compact Hausdorff topologies, i.e. compact Hausdorff algebras*. To extablish a context for our main result we would like to present two questions that have been studied in the field of topological algebra.*

Question 1 Given a variety $\mathcal{V}$ of algebras, how do the topological structure and the algebra structure interact on a (compact Hausdorff) topological algebra in $\mathcal{V}$?

For instance, using compactness and continuity of $\wedge$ one can prove that compact Hausdorff semilattices must be complete. Moreover, on compact Hausdorff lattices, the topology is uniquely determined by the lattice order, and as a corollary, a lattice homomorphism $f\colon \mathbb{L} \to \mathbb{M}$ between compact Hausdorff lattices is continuous iff it preserves all meets and all joins [Lawson1973, CCL1980, Johnstone1982].

The second question can be motivated by the folklore fact that every Stone space is homeomorphic to a projective limit of finite sets (viewed as discrete topological spaces) [Johnstone1982, RiZa2000].

Question 2 Given a variety of algebras $\mathcal{V}$, is every Stone-topological algebra $\mathbb{A}$ in $\mathcal{V}$ isomorphic to a projective limit of finite algebras in $\mathcal{V}$?

We call such projective limits of finite algebras profinite algebras*. This question has arisen in several different fields of algebra (see Notes of [Johnstone1982, Section VI-2]). Sufficient conditions for a positive answer to Question 2 are for $\mathcal{V}$ to have equationally definable principal congruences, or for $\mathcal{V}$ to be finitely generated [CDFJ2004].*

The answers to these two questions become entwined if one considers Boolean algebras. The reason for that lies in the fact that it follows from a central result in Pontryagin duality that every compact Hausdorff Boolean algebra is Stone (i.e. is zero-dimensional) [Strauss1968]. As regards the second question, it is well-established that every Stone Boolean algebra is profinite [Numakura1957]. Moreover, these categories of Boolean algebras can also be characterized lattice-theoretically as complete atomic Boolean algebras. We summarize this below.

Fact*([Lawson1973, Johnstone1982]) Pro-*$\mathbf{BA}_f \simeq \mathbf{StoneBA} = \mathbf{KHausBA} \simeq \mathbf{CABA}$.

Below we will see that if we consider modal algebras [Venema2007] instead of Boolean algebras then the above equivalences no longer hold. By **KFr** *we denote the category of Kripke frames and p-morphisms. Additionally, by* **ImFKFr** *we denote the full subcategory of image-finite Kripke frames, i.e. those frames in which each state has only finitely many successors. Our main result is the following duality:*

Theorem $\mathbf{KHausMA} \simeq \mathbf{ImFKFr}^{op}$.

Recall that **KFr** *is dually equivalent to* **CAMA***, the category of complete atomic modal algebras with completely additive modal operators and complete homomorphisms [Thomason1975]. Combined with the duality for profinite modal algebras [Vosmaer2006], this leads to the following result.*

Corollary *Pro-*$\mathbf{MA}_f \subseteq \mathbf{StoneMA} = \mathbf{KHausMA} \subseteq \mathbf{CAMA}$*, where both inclusions are full and strict.*

- *[BurSan2000] Stanley N. Burris and H.P. Sankappanavar. A Course in Universal Algebra: the Millenium Edition. Available online at http://www.math.uwaterloo.ca/ snburris/htdocs/.*
- *[CDFJ2004] David M. Clark, Brian A. Davey, Ralph S. Freese, and Marcel Jackson. Standard topological algebras: syntactic and principal congruences and profiniteness.* Algebra Universalis*, 52:343–376, 2004.*
- *[CCL1980] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M. Mislove, and D.S. Scott.* A Compendium of Continuous Lattices. *Springer-Verlag, 1980.*
- *[Johnstone1982] Peter T. Johnstone.* Stone spaces. *Cambridge University Press, 1982.*
- *[Lawson1973] Jimmie D. Lawson. Intrinsic topologies in topological lattices and semilattices.* Pacific Journal of Mathematics*, 44:593–602, 1973.*
- *[Numakura1957] Katsumi Numakura. Theorems on Compact Totally Disconnected Semigroups and Lattices.* Proceedings of the American Mathemathematical Society*, 8:623–626, 1957.*
- *[RiZa2000] Luis Ribes and Pavel Zalesskii.* Profinite Groups. *Springer, 2000.*
- *[Strauss1968] Dona Papert Strauss. Topological lattices.* Proceedings of the London Mathematical Society*, 18:217–230, 1968.*
- *[Thomason1975] S.K. Thomason. Categories of frames for modal logic.* The Journal of Symbolic Logic*, 40:439–442, 1975.*
- *[Venema2007] Yde Venema. Algebras and Coalgebras. In P. Blackburn et al., editors,* Handbook of Modal Logic. *Elsevier, 2007.*
- *[Vosmaer2006] Jacob Vosmaer. Connecting the profinite completion and the canonical extension using duality. Master's thesis, University of Amsterdam, November 2006.*

Weakly pseudocompact frames

Joann Walters-Wayland
21521 Rushford Drive, Lake Forest CA92630
`joanne@waylands.com`
Coauthors: T.Dube

A completely regular space is pseudocompact iff it is G948;-dense in each of its compactifiactions. An obvious weakening of this is to require that this holds for some, not all, compactifications. Pseudocompactness in frames has been well explored and has an analogous result: a completely regular frame is pseudocompact iff every compactification is coz-codense (a map is coz-codense if the only cozero mapped to the top *is the top itself). Consequently, a similar weakening may be observed. We explore these types of frames and give some additional characterizations in terms of strong inclusions, uniformities and cozero bases these are internal characterizations and are not translated from spatial results.*

Quandles and the Constraint Satisfaction Problem

Japheth Wood
Bard College MAT Program
`jwood@bard.edu`
Coauthors: Robert McGrail, Bard College

A Quandle is a universal algebraic knot invariant. The computational classification of finite quandles via the Constraint Satisfaction Problem has been the focus of investigation of the ASC lab, an undergraduate research program at Bard College. In this brief talk, we survey the results from the previous academic year and outline research directions for the future.

Results on Boolean algebras with applications to the model theory of l-groups

Brian Wynne
Bard College at Simon's Rock
`bwynne@colgate.edu`

In [1] Tarski gives numerical invariants that classify Boolean algebras with respect to elementary equivalence and yield the decidability of the theory of Boolean algebras. In [3] Weispfenning obtains invariants and decidability for certain l-groups by reducing their theory to that of Boolean algebras. We discuss analogous results for a different but related class of l-groups of the form $C(X)$ with X an essential P-space. The theory of such an l-group may often be reduced to that of an associated Boolean algebra with distinguished ideal. Using work of Touraille [2] we obtain invariants and decidability for some of these associated structures and hence for the corresponding l-groups.

References*:*

[1] A. Tarski, Arithmetical classes and types of Boolean algebras, Bulletin of the American Mathematical Society **55** *(1949), 64.*

[2] A. Touraille, Théories d'algèbres de Boole munies d'idéaux distingués. I. Théories élémentaires, The Journal of Symbolic Logic **52** *(1987), no. 4, 1027-1043.*

[3] V. Weispfenning, Model Theory Of Abelian l-Groups, in: A.M.W. Glass and W.C. Holland, editors, Lattice-ordered groups, *Kluwer Academic Press, Dordrecht (1989), 41-79.*

Subframe logics and NNIL-formulas

Fan Yang
Institute for Logic, Language and Computation, University of Amsterdam
`alligatoryangcn@yahoo.com`

In this talk, we investigate intuitionistic NNIL-formulas, intuitionistic subframe logics and their corresponding algebras, Brouwerian semilattices.

NNIL-formulas are the formulas that have no nesting of implications to the left. A. Visser, D. de Jongh, J. van Benthem and G. Renardel de Lavalette [1995] showed that NNIL-formulas are exactly those formulas that are preserved under taking submodels. In the talk, we give an alternative proof of this fact.

It follows from the work by Zakharyaschev [1989][1996], and from recent results by G. Bezhanishvili and S. Ghilardi [2007] that intuitionistic subframe logics are the ones that are axiomatized by $[\wedge, \to]$*-formulas, which contain only* $\wedge$ *and* $\to$ *as connectives. Noting that NNIL-formulas being preserved under submodels are preserved under subframes as well, N. Bezhanishvili [2006] succeeded in showing that NNIL-formulas are also sufficient to axiomatize subframe logics. In this talk, we provide further connections between the two ways to axiomatize subframe logics. Among other things we give a direct syntactic transformation of NNIL-formulas into frame-equivalent* $[\wedge, \to]$*-formulas.*

Furthermore, for both the variety of Heyting algebras and the variety of Brouwerian semilattices, we study the congruence extension property and the consequent property that $\mathbf{HS} = \mathbf{SH}$*, where* $\mathbf{H}$ *and* $\mathbf{S}$ *are respectively the operators of taking homomorphic images and subalgebras of the corresponding algebras. We relate the* $\mathbf{HS} = \mathbf{SH}$ *property to the dual property for intuitionistic frames concerning generated subframes and p-morphic images. For the Brouwerian semilattices no such a nice duality is known. One fact which we show in this connection is the following. Let* $\mathbf{H}^{[\wedge,\to]}$ *be the Heyting homomorphism that preserves* $\wedge$ *and* $\to$ *only. We give an example which shows that the operators* $\mathbf{H}^{[\wedge,\to]}$ *and* $\mathbf{S}$ *do not commute.*

Non-normality points

Lynne Yengulalp
University of Kansas
`spencel1@math.ku.edu`

A point x *in a normal space* X *is called a non-normality point of* X *if* $X \setminus \{x\}$ *is not normal. In this talk, I will discuss non-normality points of* $\beta X \setminus X$ *when* X *is a metric space.*