Cozero frames and perfectly regular frames

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Abstract:

Cozero elements of a frame play an important role in point-free topology. The set of cozero elements, $\mathsf{Coz}\ L$, of a frame L is a sub σ-frame of L (that is, a sublattice closed under countable suprema and finite infima). Moreover, the lattice $\mathsf{Coz}\ L$ join generates the frame L if and only if L is completely regular (a result analogous to the classical one for completely regular topological spaces).

Within the setting of completely regular frames, we consider the Dedekind-MacNeille completion of the lattice of cozero elements, which in this context coincides with the Bruns-Lakser construction. The frame obtained through this process turns out to be a sublocale of the original frame, specifically, the smallest sublocale containing $\mathsf{Coz}\ L$. A central aim of the talk is to compare this sublocale with the original frame and to present results and examples showing when the two coincide and when they differ.

Viewing $\mathsf{Coz}\ L$ both as a join generating sublattice and through its completion leads naturally to two related but distinct classes of frames: cozero frames and perfectly regular frames. We will examine the relationship between these notions and provide illustrative examples.

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