Talk #9: Maltsev classification

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3. A 'rare area' term for a variety $\mathcal{V}$ is a term $t(w, x, y, z)$ such that

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Given a set Σ of height-1 identities in the language $\mathcal{L}$, $\mathcal{V}$ satisfies Σ as a Maltsev condition if $\mathcal{V}$ has a term $s^{\mathcal{V}}$ for each basic operation symbol s of $\mathcal{L}$ and $\mathcal{V} \models s^{\mathcal{V}} \approx t^{\mathcal{V}}$ for each identity $s \approx t$ in Σ.

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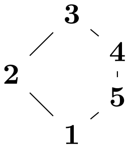

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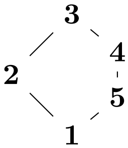

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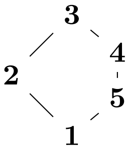

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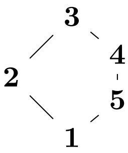

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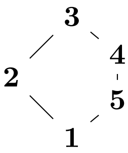

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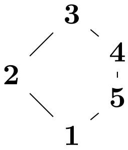

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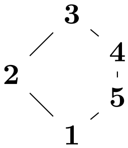

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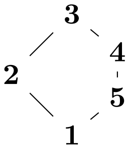

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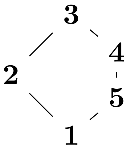

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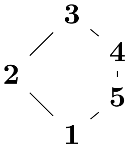

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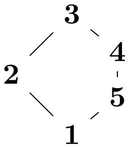

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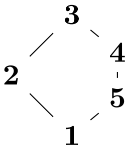

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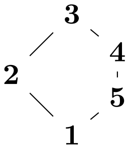

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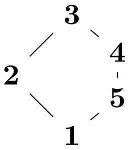

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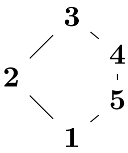

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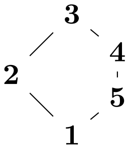

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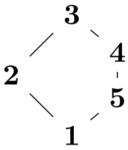

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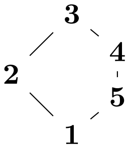

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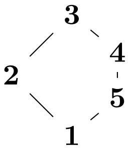

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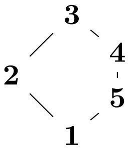

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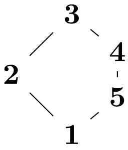

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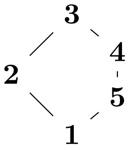

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