The following is a list of problems I consider midterm-worthy. This list of problems should serve as a good place to start studying, and it should not be considered a comprehensive list of problems from the sections we’ve covered. YOU are responsible for studying all the sections to be covered on the midterm.

1. Let $X_1, \ldots, X_n$ be a random sample of size $n$ from a uniform population on $[0, \beta]$ (i.e. $\alpha = 0$). Show that $X_{(n)}$ is a consistent estimator of $\beta$.

2. A single observation of a random variable having a uniform density with $\alpha = 0$ is used to test the null hypothesis $\beta = \beta_0$ against the alternative hypothesis $\beta = \beta_0 + 2$. If the null hypothesis is rejected if and only if the random variable takes on a value greater than $\beta_0 + 1$, find the probabilities of type I and type II errors.

3. To estimate the average time required for certain repairs, an automobile manufacturer had 40 mechanics, a random sample, timed in the performance of this task. If it took them on the average 24.05 minutes with a standard deviation of 2.68 minutes, what can the manufacturer assert with 95% confidence about the maximum error if he uses $\bar{x} = 24.05$ minutes as an estimate of the actual mean time required to perform the repairs.

4. Let $X$ be a binomial random variable with parameters $(n, \theta)$. Show that $X/n$ is a biased estimator of the parameter $\theta$. Is this estimator asymptotically unbiased?

5. Given a random sample of size $n$ from a gamma population, use the method of moments to obtain formulas for estimating the parameters $\alpha$ and $\beta$.

6. Given a random sample of size $n$ from a gamma population with known parameter $\alpha$, find the maximum likelihood estimator for $\beta$.

7. For large $n$, the sampling distribution of $S$ is sometimes approximated with a normal distribution having mean $\sigma$ and variance $\sigma^2/2n$. Show that this approximation leads to the following $(1 - \alpha)100\%$ confidence interval for $\sigma$:

$$\frac{s}{1 + \frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1 - \frac{z_{\alpha/2}}{\sqrt{2n}}}.$$ 

8. A random sample of size $n$ from an exponential population is used to test the null hypothesis $\theta = \theta_0$ against the alternative hypothesis $\theta = \theta_1 > \theta_0$. Use the Neyman–Pearson lemma to find the most powerful critical region of size $\alpha$.

9. Given a random sample of size $n$ from a uniform population with $\alpha = 3$, use the method of moments to obtain a formula for estimating the parameter $\beta$. 

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