Written Problems

1. Use the limit definition of the partial derivatives to find \( f_x(3, 2) \) and \( f_y(3, 2) \) for the function 
\[
f(x, y) = \frac{x^2}{y + 1}.
\]

2. Let \( f(x, y) \) be a function of two variables.

   (a) If \( f_x(a, b) \) or \( f_y(a, b) \) is non-zero, use local linearization to show that an equation of the line tangent to the contour of \( f \) at \((a, b)\) is 
   \[
f_x(a, b)(x - a) + f_y(a, b)(y - b) = 0.
   \]

   (b) Find the slope of the tangent line if \( f_y(a, b) \neq 0 \).

   (c) Find an equation for the tangent line to the contour of \( f(x, y) = x^2 + xy \) at \((3, 4)\).

3. Let \( f \) be a differentiable function of one variable. Show that all tangent planes to the surface 
\[
z = xf\left(\frac{y}{x}\right)
\]
intersect at a common point.

Presentation Problems

4. Define \( f(x, y) = \left(\int_3^x e^t^2 dt\right)y \). Find the directional derivative of \( f \) at the point \((3, 1)\) in the direction of the vector \( \vec{v} = (2, 3) \).

5. Let \( k > 0 \). Show that the volume in the first octant, bounded by the coordinate planes and any tangent plane of \( xyz = k \), \( x, y > 0 \) depends only on \( k \). (Hint: For \( a, b, c > 0 \), the volume in the first octant under the plane \( \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \) is \( abc/3! \).)

6. (a) Let \( z = f(x, y) \) and \( z = g(x, y) \) be differentiable surfaces. Show that if \( \nabla f \cdot \nabla g = -1 \) at a point of intersection, then the surfaces are perpendicular at that point.

   (b) Show that the surfaces \( z = \frac{1}{2}(x^2 + y^2 - 1) \) and \( z = \frac{1}{2}(1 - x^2 - y^2) \) are perpendicular at all points of intersection.