Calculus I Final Review

1. Find an equation for an exponential function through the points (1, 5), (3, 12) and with horizontal asymptote \( y = 0 \). Find another equation when the asymptote is \( y = 20 \).

2. Suppose the half-life of a radioactive isotope is 125 days. How many days will it take for a sample of the substance to reach 10% of its starting amount.

3. A sinusoid has a minimum at \((32, -2)\) and a maximum at \((38, 8)\) with no critical points in between. Find two equations for this function, one in terms of \(\sin\) and the other in terms of \(\cos\).

4. Let \( f(x) = \begin{cases} x^3 - 2x^2 + 3x - 2 & \text{if } x \leq c \\ 2x - 2 & \text{if } x > c \end{cases} \) for some constant \(c\). For what value(s) of \(c\) is \(f\) continuous? Differentiable?

5. Evaluate the following:
   
   (a) \( \lim_{x \to \infty} \frac{\ln x}{8x^2 + 1} \)
   
   (b) \( \lim_{x \to 0^+} (\sin x)^{\ln x} \)
   
   (c) \( \lim_{x \to 1} (\ln x)^{\ln x} \)
   
   (d) \( \lim_{x \to \ln 2} \frac{\sinh x - \frac{3}{4}}{x - \ln 2} \)
   
   (e) \( \lim_{x \to \infty} x - \sqrt{x^2 + x} \)
   
   (f) \( \lim_{x \to 0} \sin \left(\frac{1}{x}\right) \)
   
   (g) \( \lim_{x \to 0} x \sin \left(\frac{1}{x}\right) \)

6. 

   \[
   \begin{array}{c|cccccc}
   t \text{ (sec)} & 0 & 1 & 2 & 3 & 4 & 5 \\
   v(t) \text{ (ft/sec)} & 40 & 32 & 26 & 22 & 20 & 19 \\
   \end{array}
   \]

   (a) Approximate the instantaneous acceleration \( t = 2 \) sec.
   
   (b) Find the average acceleration over the first 4 seconds.
   
   (c) Find an upper/lower bound for the total distance traveled over the 5 seconds.
   
   (d) Suppose \( s(0) = 15 \) ft. Approximate \( s(t) \) at each second.

7. Using the limit definition, find the derivative of \( f(x) = \frac{1}{x^2 + 1} \).

8. Let \( V(t) \) be the volume of a growing yam, where \( t \) is measured in days and \( V(t) \) in cm\(^3\). Interpret the following, with units:

   (a) \( V(40) = 22 \)
   
   (b) \( V'(35) = .8 \)
   
   (c) \( V^{-1}(25) = 60 \)
   
   (d) \( (V^{-1})'(22) = 1.25 \)
   
   (e) \( \int_{10}^{20} V'(t) \, dt = 11 \)
   
   (f) Also, using the above, evaluate \( V'(40) \).
9. Suppose \( f \) is a decreasing, continuous, concave down function with \( f(0) = 6 \) and \( f'(0) = -2 \). How many zeroes can \( f \) have and where can they occur? Why? Also, can \( f(-2) = 12 \)? \( f(-2) = 4 \)? \( f(-2) = 8 \)? \( f(-2) = 10 \)?

10. Find the tangent line to \( xy^2 + y^3 = x^2 + 8 \) at \((0,2)\).

11. Verify that \( \sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y \).

12. Approximate \( \ln(1.02) \) without a calculator. Is this an over or under approximation?

13. Let \( f(x) = 3x^4 - 4x^3 + 6 \). On what intervals is \( f \) increasing/decreasing? Concave up/down? Find all critical points and any local max/mins. Find the global max/mins on \([-2,2]\).

14. Find the point on the parabola \( y = x^2 \) that is closest to \((2, \frac{1}{2})\).

15. A cylinder is changing size, but is keeping the same volume, \( 100\pi \) cm\(^3\). At some point, the height is 25 cm and the radius is decreasing at a rate of 3 cm/min. At what rate is the height changing at this moment?

16. Consider the graph defined by \( (x, y) = (e^t, 5e^{2t}) \).
   (a) Find the parametric equations for the tangent line at \( t = \ln 3 \).
   (b) Find the equation of the tangent line expressing \( y \) as a function of \( x \).
   (c) Find the speed at \( t = \ln 3 \).
   (d) Is this graph the same as \( (x, y) = (t, 5t^2) \)? as \( (x, y) = (t^2, 5t^4) \)? as \( (x, y) = (\frac{1}{t}, \frac{5}{t^2}) \), \( t > 0 \)?

17. Find the area between \( y = x^2 \) and \( y = x^3 \).

18. Suppose \( f \) is even, \( \int_2^3 f(x) \, dx = 5 \), and \( \int_{-3}^0 f(x) \, dx = 2 \). Evaluate \( \int_0^5 f(x - 3) - 2 \, dx \).

19. How does \( \left| \int_a^b f(x) \, dx \right| \) compare to \( \int_a^b |f(x)| \, dx \)?

20. Find a number \( c \) such that the average value of \( f(x) = \frac{1}{x} \) on \([c, 2c]\) is 1.

21. A rock on another planet falls from a height of 100 m and hits the ground after 5 seconds. What is the acceleration due to gravity on the planet?

22. Solve the initial value problem \( \frac{dy}{dx} = e^x + 4 \sin x \), \( y(0) = 2 \).

23. Define \( F(x) = \int_2^x \frac{1}{t^2 + 1} \, dt \). Find \( F(\sqrt{2}) \), \( F'(\sqrt{2}) \), and \( F''(\sqrt{2}) \).