Math 3130 - Assignment 3
Due February 5, 2016
Markus Steindl

(19) Show that the following maps are not linear by giving concrete vectors for which the defining properties of linear maps are not satisfied.

(a) \( g : \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} xy \\ y \end{bmatrix} \)

(b) \( h : \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} |x| + |y| \\ 2x \end{bmatrix} \)

(20) [1, Section 1.8, Ex 24] An affine transformation \( T : \mathbb{R}^n \to \mathbb{R}^m \) has the form \( T(\mathbf{x}) = A\mathbf{x} + \mathbf{b} \) with \( A \) an \( m \times n \)-matrix and \( \mathbf{b} \in \mathbb{R}^m \). Show that \( T \) is not a linear transformation if \( \mathbf{b} \neq \mathbf{0} \).

(21) Let \( T : \mathbb{R}^2 \to \mathbb{R}^3 \) be a linear map such that

\[
T\left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, T\left( \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}
\]

(a) Use the linearity of \( T \) to find \( T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) \) and \( T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) \).

(b) Determine \( T(\begin{bmatrix} x \\ y \end{bmatrix}) \) for arbitrary \( x, y \in \mathbb{R} \).

(22) Give the standard matrices for the following linear transformations:

(a) \( T : \mathbb{R}^2 \to \mathbb{R}^3, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2x + y \\ x \\ -x + y \end{bmatrix} \)

(b) the function \( S \) on \( \mathbb{R}^2 \) that scales all vectors to half their length.

(23) Give the standard matrix for the linear map \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) that rotates points (about the origin) by \( 60^\circ \) counterclockwise and then reflects them on the \( x \)-axis.

(24) Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be the reflection at the line \( 2x + 3y = 0 \). Note that \( T \) is linear.

(a) What is the reflection of the normal vector \( \mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \) of the line? What is the reflection of the vector \( \mathbf{b} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \), which is on this line? Make a drawing if necessary.

(b) Write the unit vectors \( \mathbf{e}_1, \mathbf{e}_2 \) as linear combinations of \( \mathbf{a} \) and \( \mathbf{b} \).

(c) Use the linearity of \( T \) to find the reflection of the unit vectors \( T(\mathbf{e}_1), T(\mathbf{e}_2) \) from \( T(\mathbf{a}), T(\mathbf{b}) \).

(d) Give the standard matrix for \( T \).

(25) Is \( T : \mathbb{R}^3 \to \mathbb{R}^2, \mathbf{x} \mapsto \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} \cdot \mathbf{x} \)

injective, surjective, bijective?

(26) [1, cf. Section 1.9, Ex 23/24] True or False? Correct the false statements to make them true.
(a) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by the images of the unit vectors in $\mathbb{R}^n$.

(b) $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $x \in \mathbb{R}^n$ is mapped onto some vector in $\mathbb{R}^m$.

(c) $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if every vector $x \in \mathbb{R}^n$ is mapped onto a unique vector in $\mathbb{R}^m$.

(d) A linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ cannot be one-to-one.

(27) Compute if possible $A + 3B, B \cdot A, A \cdot B, A \cdot C, C \cdot A$

for the matrices

\[
A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 1 & 3 \end{bmatrix}.
\]

If an expression is undefined, explain why.

REFERENCES