(10) [1, Section 1.4, Ex 17] How many rows of $A$ contain a pivot position? Does the equation $Ax = b$ have a solution for each $b \in \mathbb{R}^4$?

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & -1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

(11) [1, Section 1.4, Ex 31] Let $A$ be a $3 \times 2$ matrix. Explain why the equation $Ax = b$ cannot be consistent for all $b \in \mathbb{R}^n$.

(12) Let $u \in \mathbb{R}^n$ be a vector and let $c, d \in \mathbb{R}$ be scalars. Show that $(c + d)u = cu + du$.

(13) [1, cf. Section 1.5, Ex 17] Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve the equations $Ax = b$ and $Ax = 0$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

(14) [1, cf. Section 1.5, Ex 11] Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve the equations $Ax = b$ and $Ax = 0$. Express both solution sets in parametric vector form.

(15) [1, Section 1.5, Ex 31] Let $A$ be a $3 \times 2$ matrix with 2 pivot positions.

(a) Does the equation $Ax = 0$ have a nontrivial solution?

(b) Does the equation $Ax = b$ have a solution for every possible $b \in \mathbb{R}^3$?

Explain your answers!

(16) [1, Section 1.7, Ex 9] Let

$$u = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \quad w = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$$

(a) For which values of $h$ is $w$ in $\text{Span}\{u, v\}$?

(b) For which values of $h$ is $\{u, v, w\}$ linearly dependent?

(17) [1, cf. Section 1.7, Ex 21] Mark each statement True or False, and justify each answer.

(a) The columns of a matrix $A$ are linearly independent if $x = 0$ is a solution of $Ax = 0$.

(b) If $\{u, v, w\}$ is linearly dependent, then each vector is a linear combination of the other two vectors.

(c) The columns of any $4 \times 5$ matrix are linearly dependent.
(d) If \( u \) and \( v \) are linearly independent, and if \( \{u, v, w\} \) is linearly dependent, then \( w \) is in the span of \( u, v \).

(18) Show the following Theorem in 2 steps: Suppose \( Ax = b \) has a solution \( p \). Then the set of all solutions of \( Ax = b \) is

\[
p + \text{NullSpace } A = \{ p + v \mid v \in \text{NullSpace } A \}.
\]

Suppose \( Ax = b \) has a solution \( p \).

(a) Show that if \( v \) is in \( \text{NullSpace } A \), then \( p + v \) is also a solution for \( Ax = b \).

(b) Show that if \( q \) is a solution for \( Ax = b \), then \( q - p \) is in \( \text{NullSpace } A \).

References