1 Short answer questions

1. (Properties of relations) Indicate which properties each relation has by writing R (reflexive), S (symmetric), T (transitive), and/or E (equivalence).
   (a) The relation \( = \) on the integers.
   (b) The relation ‘has the same sign’ on the non-zero integers.
   (c) The relation \( \subseteq \) on the power set of a set.
   (d) The relation \( \not\subseteq \) on the power set of a set.

2. (Relations – definitions, ordered pairs, arrow diagrams) Let \( A = \{1, 2, 3, 4\} \).
   Let \( R = \{(a, b) \in A \times A : b - a \text{ is even}\} \). Draw the arrow diagram and write out the relation as a set of ordered pairs.

3. (Logical laws) Use logical laws to simplify the following expression until each variable appears at most once and there is at most one ‘\( \neg \)’ symbol.
   \[ ((\neg R) \lor P) \land (P \lor (\neg Q)) \]

4. (Quantifiers) Which of the following are true?
   (a) \( \exists x \in \mathbb{R}, x \text{ is an odd integer} \)
   (b) \( \forall x \in \mathbb{R}, x^2 + 2x + 1 = (x + 1)^2 \)
   (c) \( \forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}, x = y \)
   (d) \( \exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy = 1 \)

5. (Negating statements) Negate the following, writing the answer with quantifiers: \( \forall x \in \mathbb{Z}, \exists z \in \mathbb{Z}, x = z + 7 \).

6. (Converse and contrapositive) Write the contrapositive and converse of the following statement: If \( x + 2 > y \), then \( y^2 = 7 \).
   Contrapositive:
   Converse:
7. (Logical equivalence) True or False.
   (a) $P \lor P$ is a tautology.
   (b) $P \land Q$ is logically equivalent to $(\neg P) \lor (\neg Q)$
   (c) $\neg (P \implies Q)$ is logically equivalent to $P \land (\neg Q)$
   (d) $(P \iff Q) \iff R$ is logically equivalent to $P \iff (Q \iff R)$

8. (Evaluation of boolean expressions, truth tables) Draw a truth table for the expression $(P \land Q) \implies (\neg Q)$. Include the intermediate columns.

9. (Self-study Sections 1.9, 1.10) Clearly state Russell’s paradox.

10. (Counting by independent choices) How many ways can you break a class of 11 students up into three disjoint sets (empty sets are ok), called ‘Awesome Team’, ‘Mediocre Team’ and ‘Annoying Team’?

11. (Counting with possible overcounting) How many ways can you break a class of 11 students up into two disjoint sets (empty sets are ok).