1. Find the radius of convergence and interval of convergence for each of these power series:

(a) \( \sum_{n=2}^{\infty} \frac{(x + 5)^n}{2^n \ln n} \)

(b) \( \sum_{n=0}^{\infty} \frac{n(x - 1)^n}{4^n} \)

(c) \( \sum_{n=0}^{\infty} n!(3x + 1)^n \)

(d) \( \sum_{n=0}^{\infty} \frac{(-2)^{n+1}x^n}{n^3 + 1} \)

(e) \( \sum_{n=1}^{\infty} \frac{\ln n x^n}{n!} \)

2. Let 

\[ f(x) = \sum_{n=1}^{\infty} \frac{(x + 4)^n}{n^2} \]

Find the intervals of convergence of \( f \) and \( f' \).

3. If \( \sum b_n(x - 2)^n \) converges at \( x = 0 \) but diverges at \( x = 7 \), what is the largest possible interval of convergence of this series? What's the smallest possible?

4. The power series \( \sum c_n(x - 5)^n \) converges at \( x = 3 \) and diverges at \( x = 11 \). What are the possibilities for the radius of convergence? What can you say about the convergence of \( \sum c_n \)? Can you determine if the series converges at \( x = 6 \)? At \( x = 7 \)? At \( x = 8 \)? At \( x = 2 \)? At \( x = -1 \)? At \( x = -2 \)? At \( x = 12 \)? At \( x = -3 \)?

5. The series \( \sum c_n(x+2)^n \) converges at \( x = -4 \) and diverges at \( x = 0 \). What can you say about the radius of convergence of the power series? What can you say about the convergence of \( \sum c_n \)? What can you say about the convergence of the series \( \sum c_n 2^n \)? What can you say about the convergence/divergence of the series at \( x = -1 \)? At \( x = -3 \)? At \( x = 1 \)? At \( x = -10 \)?

6. Say that 

\[ f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \]

Find \( f'(x) \) by differentiating termwise.

7. Use any method to find a power series representation of each of these functions, centered about \( a = 0 \). Give the interval of convergence (Note: you should be able to give this interval based on your derivation of the series, not by using the ratio test.)

(a) \( \frac{1}{1 + x} \)

(b) \( \frac{1}{1 + x^2} \)

(c) \( \arctan x \)

(d) \( xe^x - x \)

(e) \( \ln(1 + x) \)
(f) \( x \ln (1 + 3x^2) \)
(g) \( \frac{\sin (-2x^2)}{x} \)
(h) \( \frac{1}{(1-x)^2} \)
(i) \( \int \frac{1}{1+x^5} \, dx \)

8. Determine the function or number represented by the following series:

(a) \( \sum_{n=1}^{\infty} nx^{n-1} \)
(b) \( \sum_{n=1}^{\infty} nx^n \)
(c) \( \sum_{n=0}^{\infty} \frac{x^{2n}}{5^{2n}n!} \)
(d) \( \sum_{n=0}^{\infty} \frac{(-1)^n 2n x^{2n+1}}{(2n+1)!} \)
(e) \( \sum_{n=1}^{\infty} \frac{x^{2n}}{n} \)
(f) \( \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{(2n)!} \)

9. A car is moving with speed 20 m/s and acceleration 2 m/s\(^2\) at a given instant. Using a second degree Taylor polynomial, estimate how far the car moves in the next second.

10. Estimate \( \int_0^1 \sin t \, dt \) using a 3rd degree Taylor Polynomial. What degree Taylor Polynomial should be used to get an estimate within 0.005 of the true value of the integral? (Hint: use the alternating series estimate).

11. Calculate the Taylor series of \( \ln(1+x) \) by two methods. First calculate it “from scratch” by finding terms from the general form of Taylor series. Then calculate it again by starting with the Taylor series for \( f(x) = \frac{1}{1-x} \) and manipulating it. Determine the interval of convergence each time.

12. Express the integral as an infinite series.
\[
\int \frac{e^x - 1}{x} \, dx
\]

13. Let \( f(x) = \frac{1}{1-x} \).

(a) Find an upper bound \( M \) for \( |f^{(n+1)}(x)| \) on the interval \((-1/2, 1/2)\).

(b) Use this result to show that the Taylor series for \( \frac{1}{1-x} \) converges to \( \frac{1}{1-x} \) on the interval \((-1/2, 1/2)\).
14. Consider the function $y = f(x)$ sketched below.

Suppose $f(x)$ has Taylor series

$$f(x) = a_0 + a_1(x - 4) + a_2(x - 4)^2 + a_3(x - 4)^3 + ...$$

about $x = 4$.

(a) Is $a_0$ positive or negative? Please explain.
(b) Is $a_1$ positive or negative? Please explain.
(c) Is $a_2$ positive or negative? Please explain.

15. How many terms of the Taylor series for $\ln(1 + x)$ centered at $x = 0$ do you need to estimate the value of $\ln(1.4)$ to three decimal places (that is, to within .0005)?

16. (a) Find the 4th degree Taylor Polynomial for $\cos x$ centered at $a = \pi/2$.

(b) Use it to estimate $\cos(89^\circ)$.
(c) Use Taylor’s inequality to determine what degree Taylor Polynomial should be used to guarantee the estimate to within .005.

17. (a) Find the 3rd degree Taylor Polynomial $P_3(x)$ for $f(x) = \sqrt{x}$ centered at $a = 1$ by differentiating and using the general form of Taylor Polynomials.
(b) Use the Taylor Polynomial in part (a) to estimate $\sqrt{1.1}$.
(c) Use Taylor’s inequality to determine how accurate is your estimate is guaranteed to be.

18. Use Taylor’s inequality to find a reasonable bound for the error in approximating the quantity $e^{0.60}$ with a third degree Taylor polynomial for $e^x$ centered at $a = 0$.

19. Consider the error in using the approximation $\sin \theta \approx \theta - \frac{\theta^3}{3!}$ on the interval $[-1, 1]$. Where is the approximation an overestimate? Where is it an underestimate?

20. Write down from memory the Taylor Series centered around $a = 0$ for the functions $e^x$, $\sin x$, $\cos x$ and $\frac{1}{1-x}$.

21. (a) Find the 4th degree Taylor Polynomial for $f(x) = \sqrt{x}$ centered at $a = 1$ by differentiating and using the general form of Taylor Polynomials.
(b) Use the previous answer to find the 4th degree T.P. for $f(x) = \sqrt{1-x}$ centered at $x = 0$.
(c) Use the previous answer to find the 3rd degree T.P. for $f(x) = \frac{1}{\sqrt{1-x}}$.
(d) Use the previous answer to find the 3rd degree T.P. for $f(x) = \sqrt{1-x^2}$.
(e) Use the previous answer to find the 3rd degree T.P. for $f(x) = \arcsin x$. 