Developing your intuition: For each of the following series, guess if it diverges, converges conditionally or converges absolutely. Keep in mind that you must answer two separate questions: 1. Does the series converge? and 2. Does the series converge absolutely? Name the test(s) you would use to answer each of these questions. Usually you are required to give a detailed solution, but for this worksheet, just briefly describe your overall strategy.

1. \[\sum_{n=1}^{\infty} \frac{(-1)^n(n + \frac{1}{2})}{n - \frac{1}{2}}\]

2. \[\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}\]

3. \[\sum_{n=1}^{\infty} \frac{2^n}{n!}\]

4. \[\sum_{n=1}^{\infty} \frac{\sin n}{n!} \frac{2^n}{n!}\]

5. \[\sum_{n=2}^{\infty} \frac{(-1)^n(n^3 + 1)}{n^4 + n - 4}\]

6. \[\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}\]

7. \[\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3 + n}\]

8. \[\sum_{n=2}^{\infty} \frac{(-1)^n \arctan n}{\sqrt{n}}\]

9. \[\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}\]

10. \[\sum_{n=2}^{\infty} \frac{(-1)^n n}{(\ln n)^2}\]
11. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^7 + n}}{\sqrt{n^9 + n^5}}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^9 + n}}{\sqrt{n^{10} + n^5}}$

13. $\sum_{n=1}^{\infty} \frac{(-1)^n 10n^2}{n^4 + 1}$

14. $\sum_{n=2}^{\infty} \frac{1}{n \sqrt{n \ln n}}$

15. $\sum_{n=1}^{\infty} \frac{n(-2)^n}{n!}$

16. $\sum_{n=1}^{\infty} \frac{2 - 5^n}{11^{n-1}(-1)^n}$

17. $\sum_{n=1}^{\infty} \sqrt{n^{2n+1}}$

18. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{4n^5 + n^4 - 1}}$

19. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n n!}{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n - 1)}$

20. $\sum_{n=1}^{\infty} \frac{(-1)^n \sin (n^2)}{2^n}$

21. $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^{n^2}}$
Answers:

1. Diverges.
   (divergence test)

2. Converges absolutely.
   (ratio test or integral test)

3. Converges absolutely.
   (ratio test)

4. Converges absolutely.
   First show \( \sum \frac{2^n}{n!} \) converges using the ratio test, then compare the absolute value of our series to \( \sum \frac{2^n}{n!} \) using term-size comparison.

5. Converges conditionally.
   Use A.S.T to show convergence. Then take the absolute value and use L.C.T. (compare to \( \sum b_n = \sum \frac{1}{n} \)) to show convergence is NOT absolute.

6. Converges absolutely.
   Compare to \( \sum \frac{1}{n^{3/2}} \) using term-size comparison.

7. Converges absolutely.
   Take absolute value, use L.C.T., and compare to \( \sum \frac{1}{n^3} \).

8. Converges conditionally.
   Use A.S.T to show convergence and L.C.T with \( \sum \frac{1}{\sqrt{n}} \) to show convergence is not absolute.

   Take absolute value, then either: compare term-wise to \( \sum \frac{1}{\sqrt{n}} \)
   or: use the integral test (integrate by parts with \( u = \ln n \)).

10. Diverges.
   (divergence test)

11. Converges conditionally.
    Use A.S.T to show convergence and then take absolute value and compare to \( \sum \frac{1}{n} \) to show that convergence is not absolute (L.C.T.).

12. Converges absolutely.
    Take absolute value, then compare to \( \sum \frac{1}{n^{3/2}} \) using limit comparison.

13. Converges absolutely.
    Either compare to \( \sum \frac{1}{n!} \) using limit comparison, or compare to \( \sum \frac{10}{n} \) using term-size comparison.

    Use integral test (integrate by substitution with \( u = \ln n \)).

15. Converges absolutely.
    (ratio test)

    (Break the difference into two separate series, each is a geometric series, \( |r| < 1 \))

17. Diverges.
    (divergence test)

18. Converges absolutely.
    Compare to \( \sum \frac{1}{n^{3/2}} \).

19. Diverges.
    (ratio test, careful with the cancellations)

20. Converges absolutely.
    Take absolute value and then compare to \( \sum \frac{1}{2^n} \).

    (ratio test)