1. In the graph of \( f(x) \) below, number lines are used to mark where \( f(x) \) is zero/positive/negative/undefined, where \( f'(x) \) is zero/positive/negative/undefined, and where \( f''(x) \) is zero/positive/negative/undefined. Closed circles on the number lines indicate a zero-value, and open-circles indicate an undefined value.

(a) What do the closed circles on the number line for \( f(x) \) correspond to on the graph of \( f(x) \)?

(b) How does each + or − sign on the number line for \( f(x) \) relate to the graph?

(c) What does the closed circle at \( x = -1 \) on the number line for \( f'(x) \) correspond to on the graph of \( f(x) \)?

(d) What does the open circle at \( x = 2 \) on the number line for \( f'(x) \) correspond to on the graph of \( f(x) \)?

(e) How does each + or − sign on the number line for \( f'(x) \) relate to the graph?

(f) What does the open circle at \( x = 2 \) on the number line for \( f''(x) \) correspond to on the graph of \( f(x) \)?

(g) How does each + or − sign on the number line for \( f''(x) \) relate to the graph?
2. For the graph of \( f(x) \) shown below, fill in the number lines for \( f(x) \), \( f'(x) \) and \( f''(x) \), marking closed circles where there is a zero, marking open circles for undefined points, and marking \(+\) and \(-\) signs on each interval to show positive/negativeness.

3. Draw a graph of \( f(x) \) that fits the information shown in the number lines.
4. The middle graph drawn below shows $f'(x)$. Using the principles you learned in the previous problem, draw a possible graph of $f(x)$ above it, and a graph of $f''(x)$ below it. (If you are stuck try drawing the graph of $f''(x)$ first.)
5. This problem investigates the derivative of the absolute value function. Recall that we define the absolute value as:

\[ |x| = \begin{cases} 
  x & \text{if } x \geq 0, \\
  -x & \text{if } x < 0. 
\end{cases} \]

(a) In the space provided, draw a graph of the function \( f(x) = |x| \).

(b) Using your graph from part (a), and your understanding of the derivative as the rate of change/slope of the tangent line, find the derivative function \( f'(x) \) of the above function \( f(x) = |x| \), for \( x \) not equal to 0 (fill in the blanks):

\[ f'(x) = \begin{cases} 
  & \text{if } x > 0, \\
  & \text{if } x < 0. 
\end{cases} \]

(c) But what about \( f'(0) \)? It is not so clear from the picture even how to draw a tangent line to the function at the origin. So let’s try to compute \( f'(0) \) by first looking at the corresponding lefthand and righthand limits of the difference quotient.

   i. Compute \( \lim_{h \to 0^+} \frac{f(0+h)-f(0)}{h} \). Hint: use the piecewise definition of \( f(x) \) given above.

   ii. Compute \( \lim_{h \to 0^-} \frac{f(0+h)-f(0)}{h} \).

   iii. What do your answers to parts (i) and (ii) tell you about \( f'(0) \)? Please explain.
6. Using what you’ve learned above, sketch the graph of a continuous function $g(x)$ such that $g(x)$ is not differentiable at $x = -1$, $x = 2$, nor $x = 3$.

7. Create a piecewise function where one piece is a quadratic function and the other piece is a linear function which is continuous everywhere but not differentiable at $x = 0$.

$$f(x) = \begin{cases} \text{_________} & \text{if } x \geq 0, \\ \text{_________} & \text{if } x < 0. \end{cases}$$

8. Create a piecewise function where one piece is a quadratic function and the other piece is a linear function which is continuous and differentiable everywhere.

$$f(x) = \begin{cases} \text{_________} & \text{if } x > 0, \\ \text{_________} & \text{if } x \leq 0. \end{cases}$$