1. Review and Warm-up: The graph of \( f \) is shown below. Calculate exactly each of the definite integrals that follow.

\[
\begin{align*}
(a) \quad & \int_0^2 f(x) \, dx = \\
(b) \quad & \int_0^3 f(x) \, dx = \\
(c) \quad & \int_1^5 f(x) \, dx = \\
(d) \quad & \int_5^8 f(x) \, dx = \\
(e) \quad & \int_2^4 f(x) \, dx = \\
(f) \quad & \int_2^8 f(x) \, dx = 
\end{align*}
\]

2. Let \( s(t) \) be the position, in feet, of a car along a straight east/west highway at time \( t \) seconds. Positive values of \( s \) indicate eastward displacement of the car from home, and negative values indicate westward displacement. At \( t = 0 \) the car is at home. Let \( v(t) \) represent the velocity of this same car, in feet per second, at time \( t \) seconds (see graph below).

(a) Write a definite integral representing each of the following:

\[
\begin{align*}
s(10) = \\
s(30) = \\
s(t) = 
\end{align*}
\]

Now use these integrals and the velocity graph to help you fill in the chart below:

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s(t) )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Use these values to help you plot the position function.
(c) Fill in the chart below:

<table>
<thead>
<tr>
<th>Definite integral of velocity</th>
<th>Change in position</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int_{0}^{10} v(t)dt =$</td>
<td>$s(10) - s(0) =$</td>
</tr>
<tr>
<td>$\int_{10}^{20} v(t)dt =$</td>
<td>$s(20) - s(10) =$</td>
</tr>
<tr>
<td>$\int_{0}^{40} v(t)dt =$</td>
<td>$s(40) - s(0) =$</td>
</tr>
</tbody>
</table>

(d) Why do these two columns give the same answers?

3. The following data is from the U.S. Bureau of Economic Analysis. It shows the rate of change $r(t)$ (in dollars per month) of per capita personal income, where $t$ is the number of months after January 1, 2012.

<table>
<thead>
<tr>
<th>$t$ (months)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(t)$ (dollars per month)</td>
<td>154</td>
<td>17</td>
<td>10</td>
<td>79</td>
<td>278</td>
<td>-432</td>
<td>290</td>
</tr>
</tbody>
</table>

Use left-hand Riemann sums to estimate the total change in personal income during 2012.

4. A can of soda is put into a refrigerator to cool. The temperature of the soda is given by $F(t)$. The rate at which the temperature of the soda is changing is given by

$$F'(t) = f(t) = -25e^{-2t} \text{ (in degrees Fahrenheit per hour)}$$

(a) Find the rate at which the soda is cooling after 0, 1, and 2 hours. Then use this information to estimate the temperature of the soda after 3 hours if the soda was $60^\circ F$ when it was placed in the refrigerator.
(b) Now we will find the exact temperature of the soda after three hours have passed.

i. Find an antiderivative of $f(t)$, that is, find a function $G(t)$ such that $F'(t) = f(t) = -25e^{-2t}$.

(Hint: take a couple of derivatives of $f(t)$ and try to find a pattern.)

ii. The Fundamental Theorem of Calculus (the Evaluation Theorem) tells us that

$$\int_{a}^{b} f(t) \, dt = F(b) - F(a).$$

Use this theorem and the function you found in the last step to find the temperature of the soda after 3 hours have passed.

(c) Why do you think your estimate in part (a) is so far off?
5. (a) Write an integral which represents the area between $f(x) = x^4$ and the $x$-axis, between \(x = 0\) and \(x = 2\).

(b) Evaluate this integral using the Fundamental Theorem of Calculus (the Evaluation Theorem).

6. (a) Using the Fundamental Theorem of Calculus (as in the last problem), evaluate \(\int_{0}^{\pi} \cos(x) \, dx\).

(b) Show the area represented by the integral in part (a) on the graph.

7. Evaluate \(\int_{0}^{1} \frac{1}{1 + x^2} \, dx\) without using technology.