Area accumulation functions and the FTC, analytical perspective

1. Let \( F(x) = \int_{3}^{x} e^{5t} dt \)

   (a) Find a formula for \( F(x) \) by anti-differentiating and substituting.

   \[ F(x) = \frac{1}{5} \left( e^{5x} - e^{15} \right) \]

   (b) Differentiate to find \( F'(x) \).

   \[ F'(x) = e^{5x} \]

   (c) Explain your result.

   In part (a) we found the area accumulated under \( e^{5t} \) between 3 and \( x \) expressed as a function of \( x \). In part (b) we found the instantaneous rate of change of area accumulation, which when we use an infinitely small change in \( x \), will be equal to the \( y \)-value, or height, of the function, which is given by \( y = e^{5x} \).

   (d) Why does the lower limit of integration not affect the derivative?

   The lower limit is constant and only shifts the area accumulation function; it does not affect its rate of change.

   (e) Using what you noticed and learned above, find \( \frac{d}{dx} \left[ -\int_{-5}^{x} \arctan t \, dt \right] \).

   \[ \cos (x) \ln (\sin x) \]

2. Let \( F(x) = \int_{4}^{x^2} \cos t \, dt \)

   (a) Find a formula for \( F(x) \) by anti-differentiating.

   \[ F(x) = \sin x^2 - \sin 4 \]

   (b) Differentiate to find \( F'(x) \).

   \[ F'(x) = 2x \cos x^2 \]

   (c) Using what you noticed and learned above, find \( \frac{d}{dx} \left[ \int_{2}^{\sin x} \ln t \, dt \right] \).

   \[ \cos (x) / \sin x \]
Find a general formula for what you discovered in the last problem:

3. If \( F(x) = \int_{0}^{g(x)} h(t) \, dt \), what is \( F'(x) \)?

4. If \( F(x) = \int_{x}^{0} h(t) \, dt \), what is \( F'(x) \)? (Hint: try an experiment on \( F(x) = \int_{x}^{0} \sec^2 t \, dt \))

5. If \( F(x) = \int_{x^2}^{1-x} \frac{t + 1}{t - 1} \, dt \), what is \( F'(x) \)?

Find a general formula for what you discovered in the last problem:

6. If \( F(x) = \int_{a(x)}^{b(x)} h(t) \, dt \), what is \( F'(x) \)?