Math 8174: Homework 7

Due April 29, 2009

1. Prove the Lower Bound Lemma: For $P : G \to \mathbb{R}_{\geq 0}$,

$$||P - \pi|| \geq \frac{1}{2|G|} \sum_{\substack{\lambda \in \hat{G} \\ \chi^\lambda \neq \mathbb{1}}} \chi^\lambda(1) \mathrm{tr}\left(\hat{P}(\lambda) \overline{\hat{P}(\lambda)}^T \right).$$

2. For each of the following probabilities $P : G \to \mathbb{R}_{\geq 0}$, the corresponding transition matrix M_P is conjugate to a diagonal matrix D. Find that D.

   (a) $G = C_r = \{0, 1, 2, \ldots, r-1\}$, r odd,

$$P(j) = \begin{cases} 1/2, & \text{if } j \in \{1, r-1\}, \\ 0, & \text{otherwise.} \end{cases}$$

   (b) $G = S_n$,

$$P(w) = \begin{cases} 2/n^2, & \text{if } w = (i,j), \\ 1/n, & \text{if } w = 1, \\ 0, & \text{otherwise.} \end{cases}$$

3. Describe (as precisely as possible) the walk on partitions implied by

$$P(w) = \begin{cases} 2/n^2, & \text{if } w = (i,j), \\ 1/n, & \text{if } w = 1, \\ 0, & \text{otherwise.} \end{cases}$$